Conventions, Notations, and Abbreviations

Charles Kusniec

October 6, 2023

C000000 Conventions, notations, abbreviations, glossary, and references.

Charles Kusniec
https://orcid.org/0009-0006-1146-7847
charleskusniec@icloud.com

Released: 28 Oct 2020 **Revised:** 3 October 2023 21:31:20

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1. Introduction.

This summary organizes the conventions, notations, abbreviations, glossary, and references adopted throughout the studies.

We will use these standards in all studies, texts, figures, tables, graphs, etc.

The first published version is the pre-print available online at https://easychair.org/publications/preprint/4ZDm.

The latest one we are currently keeping available online at https://www.facebook.com/groups/snypo/posts/653023753021234/.

2. Sequences notation.

In general, we will try to give sequences the same notation that OEIS (https://oeis.org/) uses as standard.

Eventually, we create the SNYPO´s notation "Cxxxxxx" as per below.

2.1. The SNYPO group on Facebook.

To help with the studies, we have created an open Facebook group called SNYPO at https://www.facebook.com/groups/snypo.

We will create a SNYPO sequence equivalent to an existing sequence already in the OEIS at https://oeis.org/ if occur any of the hypotheses below:

- There is a characteristic not yet reported in the OEIS.
- It is necessary to show a different range or any important missing element(s) of the data sequence. For example, the symmetry point in polynomial sequences.
- There is a need to show photos, links, or other references not yet reported or clarified in the OEIS.
- There are two or more different sequences reported in the OEIS, which are the same sequence of different offsets, and/or different directions.

SNYPO accepts any number sequence that has a formula or algorithm.

Also accepts a sequence of sequences in any dimension and any combination.

The elements of the sequences are the numbers in base 10, unless a necessary note specifying the different base.

Numbers can be integers, rational, irrational, real, complex and/or a mixture of these.

If there is a mathematical algorithm behind, then some sequences may have words and/or letters like true, false, even, odd, etc.

2.2. Minimum number of elements of a sequence.

In SNYPO we can create the "empty sequence". See at C002000 https://www.facebook.com/groups/snypo/posts/419165166407095/.

With one element or any constant sequence, we construct the 0th degree polynomial integer sequence.

With two elements we construct the 1st degree polynomial integer sequence.

So, SNYPO does not have a general limitation on the minimum number of elements in a sequence.

The only constraint is that there must be a mathematical logic or an algorithm that justifies the sequence.

The minimum number of elements in a sequence must be at least the minimum necessary to follow the algorithm or the logic described.

2.3. Notation for polynomials in our studies.

We are adopting the following criteria:

- If not mentioned otherwise, whenever we refer to polynomials, we are referring to polynomials of one variable. We will also call this variable the index.
- We reserve the use of parentheses () only in the equation formulas.
- To express the functions and derivatives, we substitute parentheses by square brackets [].
- To denote data sequences, we use curly brackets $\{\}$. The minimum finite set of elements that produces a polynomial sequence are denoted between curly brackets.
- To differentiate from finite sequences, infinite data sequences begin and/or end with the three dots...
- Generically, we denote any polynomial function element as being $Y[y]$.
    - The reason to use $Y[y]$ is because when we want to draw the polynomial in the XY plane (like GeoGebra), we make x in the function of y, or $x = Y[y]$.
- When we want to distinguish or highlight the d^{th} degree of the polynomial, we note $Yd[y]$ or $x = Yd[y]$.
- When we want to make the p^{th} power operation on a d^{th} degree polynomial:
    - if we want to mention the degree d of the polynomial: $(Yd[y])^p$
    - if we do not want to mention the degree d of the polynomial: $(Y[y])^p$
- Notation for derivatives:
    - example for 3rd derivative:

$$Y'''[y] = Y^{[3]}[y] = \frac{d^3Y[y]}{dy^3}$$

    - example for nth derivative:

$$Y^{[n]}[y] = \frac{d^nY[y]}{dy^n}$$

2.4. The generic polynomial equation.

Usually, the generic equation of d^{th} degree polynomial is:

$$x = Yd[y] = a_d y^d + a_{d-1}y^{d-1} + \cdots + a_4 y^4 + a_3 y^3 + a_2 y^2 + a_1 y + a_0$$

Because our studies use quadratics a lot, if we do not make any observations, we will represent the generic polynomial equation as:

$$x = Yd[y] = a_d y^d + a_{d-1}y^{d-1} + \cdots + a_4 y^4 + a_3 y^3 + ay^2 + by + c$$

$$a = a_2$$

$$b = a_1$$

$$c = a_0$$

The nomenclature used to express the number of elements needed to form a polynomial is the same nomenclature used in music to define the number of elements in a band: solo, duet, trio, quartet, etc.

One element, the sole element, defines 0^{th} degree polynomial. We express the solo elements as:

$$x = Y0[y] = c \equiv \{x_1\}$$

Two consecutive elements, a duet, define 1^{st} degree polynomial. We express the duet elements as:

$$x = Y1[y] = by + c \equiv \{x_1, x_2\}$$

(Note: If we had used parentheses instead of curly brackets, then there could have been confusion with the point notation in the XY plane.)

Three consecutive elements, a trio, define 2^{nd} degree polynomial. We express the trios as:

$$x = Y2[y] = ay^2 + by + c \equiv \{x_1, x_2, x_3\}$$

Four consecutive elements, a quartet, define 3^{rd} degree polynomial. We express the quartets as:

$$x = Y3[y] = a_3y^3 + ay^2 + by + c \equiv \{x_1, x_2, x_3, x_4\}$$

Five consecutive elements, a quintet, define 4^{th} degree polynomial. We express the quintets as:

$$x = Y4[y] = a_4y^4 + a_3y^3 + ay^2 + by + c \equiv \{x_1, x_2, x_3, x_4, x_5\}$$

Six consecutive elements, a sextet, define 5^{th} degree polynomial. We express the sextets as:

$$x = Y5[y] = a_5y^5 + a_4y^4 + a_3y^3 + ay^2 + by + c \equiv \{x_1, x_2, x_3, x_4, x_5, x_6\}$$

This continues for sextic, septic, octic, nonic, decic, etc.

Following the study "Shift, Symmetry and Asymmetry in Polynomial Sequences"[20], see in the table below which elements must be used to form the simplest equation at offset $f = 0$ according to the degree of the polynomial.

Index y	Polynomial $Yd[y]$	Elements $x = Yd[y]$	$d = 0$ Sole	$d = 1$ duet	$d = 2$ trio	$d = 3$ quartet	$d = 4$ quintet	$d = 5$ sextet	$d = 6$
-3	$Yd[-3]$	e							x_1
-2	$Yd[-2]$	f					x_1	x_1	x_2
-1	$Yd[-1]$	g			x_1	x_1	x_2	x_2	x_3
0	$Yd[0]$	h	x_1	x_1	x_2	x_2	x_3	x_3	x_4
1	$Yd[1]$	i		x_2	x_3	x_3	x_4	x_4	x_5
2	$Yd[2]$	j				x_4	x_5	x_5	x_6
3	$Yd[3]$	k						x_6	x_7

C000000 Figure 1 Table.

The letters from "e" to "k" represent the elements $x = Yd[y]$ for $-3 \leq y \leq 3$.

3. The SOM numbers.

We will call the SOM numbers the union of the https://oeis.org/A000290 square number UNION https://oeis.org/A002378 oblong number UNION https://oeis.org/A005563 (square–1) number.

During these studies, we found that these three types of numbers follow the entire number line in a repetitive and uniform manner with no omissions or repetitions. Thus, these numbers are the reference points for our analysis of the number line.

The first three positive SOM numbers are {1,2,3}.

3.1. Oblong numbers.

In our studies, we will call the function of y, $Y[y] = y^2 - y$ as the oblong function or oblong function at offset $f = 0$ ($oblong = oblong_{f=0}$).

We will call the function of y, $Y[y] = y^2 + y$ as the oblong function at offset $f = -1$ ($oblong_{f=-1}$).

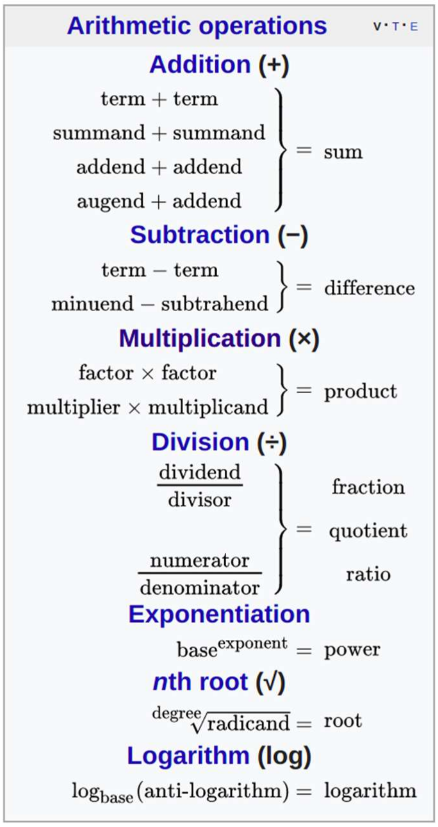
Both $Y[y] = y^2 \pm y$ have the same sequence of integers catalogued in the OEIS as https://oeis.org/A002378.

In general, we call the function $Y[y] = y^2 - y + c$ as (oblong plus a constant c) numbers. In the case of $c = -1$, the function $Y[y] = y^2 - y - 1$ produces the sequence of integers in the form of (oblong minus 1) numbers catalogued in the OEIS as https://oeis.org/A165900.

4. Factor vs. Divisor (disambiguation).

Because of the discussion that took place in https://www.mersenneforum.org/showthread.php?t=28810 from post #33 https://www.mersenneforum.org/showpost.php?p=636399&postcount=33 forward, let us define what is a factor and what is a divisor of an integer.

Let us use the English language definitions mentioned in https://en.wikipedia.org/wiki/Multiplication and https://math.stackexchange.com/questions/4367999/for-commutative-arithmetic-why-do-we-have-asymmetrical-nomenclature-like-mul. Here is the picture:



C002800 Figure 2 Reproduction of the figure used by Wikipedia about arithmetic operations available online at https://en.wikipedia.org/wiki/Multiplication.

Then we have:

$$factor1\ X\ factor2 = multiplier\ X\ multiplicand = product$$

$$\frac{dividend}{divisor} = \frac{numerator}{denominator} = fraction = quotient = ratio$$

Whenever we talk about factors or divisors, we are referring to integers. Like this:

$$\boxed{\begin{gathered} factor1\ X\ factor2 = integer1\ X\ integer2 \\ \frac{dividend}{divisor} = \frac{integer1}{integer2} \end{gathered}}$$

Then,

$$\boxed{\begin{gathered} multiplier\ X\ multiplicand = complex\ number1\ X\ complex\ number2 \\ \frac{numerator}{denominator} = \frac{complex\ number1}{complex\ number2} \end{gathered}}$$

That way, whenever we mention a product resulting from multiplication between integers, there will only be factors being multiplied.

Every product is the result of the multiplication of two or more factors. We won't say that a product is the result of multiplying n-divisors without mentioning that those n-divisors are now n-factors.

The only possibility of there being a divisor being multiplied by another number to result in a product is calculating the rest of the division.

Whenever we talk about integer division with rest 0, the integer denominator will always be a divisor. Never a factor. We won't say that an integer quotient with rest 0 is the result of dividend divided by a factor without mentioning that this factor is now a divisor.

For a two factors multiplication to produce the desired integer, then the two factors to be multiplied necessarily need to be two complementary divisors.

4.1. The use of Factor vs. Divisor.

Although in some cases the divisors of an integer are also the factors of it and vice-versa, this is not always true. For example, the composite number 6 has four positive divisors: $\{1,2,3,6\}$. But we cannot say that number 6 has 4 positive factors.

Number 36 may have four factors $36 = 1 * 2 * 3 * 6$, or three factors $36 = 1 * 6 * 6 = 1 * 3 * 12$, or two factors $36 = 1 * 36$. It becomes complex if we think of a definition of the number 36 using only the factors or only the multiplication.

But if we think only of the division, it is straight to the definition of any integer. Number 36 has exactly nine positive divisors: $\{1,2,3,4,6,9,12,18,36\}$. No other number has these exact 9 divisors set.

But if we transform these divisors into factors then we can produce several other products besides the number 36. We've lost control. That´s why the set of distinct positive and/or negative divisors completely and conclusively defines the primality of any integer.

Note how the use of distinct positive divisors from integer 36 avoids listing divisors 6 or 1 more than once.

When we talk about factors, repetitions of factors in composites mostly occur: $36 = 6 * 6 = 2 * 2 * 3 * 3 = 1 * 1 * 1 * \ldots * 1 * 36$.

The number of factors is not conclusive to define the primality of the integers. This is because we can always express all integers as each being the multiplication of only two factors.

The number of divisors is conclusive to define the primality of the integers. Especially when we use the concept of the pair of complementary divisors.

5. The golden ratio numbers (aureum numbers).

We will consider:

$$PHI = \Phi = \frac{\sqrt{5}+1}{2} = \sqrt{\frac{5}{4}+\frac{1}{2}} = \sqrt{1.25}+0.5 = \frac{1}{phi} = \frac{1}{\varphi} = 1.618033988749894\ldots$$

$$PHI^2 = \Phi^2 = \left(\frac{\sqrt{5}+1}{2}\right)^2 = \frac{\sqrt{5}+3}{2} = \sqrt{1.25}+1.5 = 1+\Phi = 1+PHI$$
$$= 2.618033988749894\ldots$$

$$2*PHI^2 = 2\Phi^2 = 2\left(\frac{\sqrt{5}+1}{2}\right)^2 = 3+\sqrt{5} = 5.236067977499789\ldots$$

$$phi = \varphi = \frac{\sqrt{5}-1}{2} = \sqrt{\frac{5}{4}-\frac{1}{2}} = \sqrt{1.25}-0.5 = \frac{1}{PHI} = \frac{1}{\Phi} = 0.618033988749894\ldots$$

$$phi^2 = \varphi^2 = \left(\frac{\sqrt{5}-1}{2}\right)^2 = \frac{-\sqrt{5}+3}{2} = -\sqrt{1.25}+1.5 = 1-\varphi = 1-phi$$
$$= 0.381966011250105\ldots$$

$$2*phi^2 = 2\varphi^2 = 2\left(\frac{\sqrt{5}-1}{2}\right)^2 = 3-\sqrt{5} = 0.763932022500210\ldots$$

C000000 Figure 3 The golden ratio numbers.

6. Notation for differences between two consecutive elements.

Let us denote the differences between two consecutive elements in any polynomial as:

$$dif_1[y] = dif = Yd[y+1] - Yd[y]$$

Then, the differences between the consecutive differences are:

$$dif_2[y] = difdif = dif_1[y+1] - dif_1[y]$$
$$dif_3[y] = difdifdif = dif_2[y+1] - dif_2[y]$$
$$dif_4[y] = difdifdifdif = dif_3[y+1] - dif_3[y]$$
$$\ldots$$
$$dif_h[y] = difdif\ldots h\ldots dif = dif_{h-1}[y+1] - dif_{h-1}[y]$$

7. Y-axis in XY plane and spreadsheet.

Because our tables show vertical sequences where the indexes are vertical and because in vertical, we have the Y-axis in the XY plane, then the elements of the sequences must appear on the X-axis depending on the Y-axis.

Because of that, in all these studies, we represent a generic polynomial equation as being in the function of y, or just the function $Y[y]$, or $x = Y[y]$.

The vertical Y-axis may have two directions:

	y	Y[y]
	20	Y[20]
	19	Y[19]
	18	Y[18]
	17	Y[17]
	16	Y[16]
	15	Y[15]
	14	Y[14]
	13	Y[13]
	12	Y[12]
	11	Y[11]
	10	Y[10]
	9	Y[9]
	8	Y[8]
	7	Y[7]
	6	Y[6]
	5	Y[5]
	4	Y[4]
	3	Y[3]
	2	Y[2]
Y[1]	1	Y[1]
Y[0]	0	Y[0]
Y[-1]	-1	Y[-1]
	-2	Y[-2]
	-3	Y[-3]
	-4	Y[-4]
	-5	Y[-5]
	-6	Y[-6]
	-7	Y[-7]
	-8	Y[-8]
	-9	Y[-9]
	-10	Y[-10]
	-11	Y[-11]
	-12	Y[-12]
	-13	Y[-13]
	-14	Y[-14]
	-15	Y[-15]
	-16	Y[-16]
	-17	Y[-17]
	-18	Y[-18]
	-19	Y[-19]
	-20	Y[-20]

	y	Y[y]
	-20	Y[-20]
	-19	Y[-19]
	-18	Y[-18]
	-17	Y[-17]
	-16	Y[-16]
	-15	Y[-15]
	-14	Y[-14]
	-13	Y[-13]
	-12	Y[-12]
	-11	Y[-11]
	-10	Y[-10]
	-9	Y[-9]
	-8	Y[-8]
	-7	Y[-7]
	-6	Y[-6]
	-5	Y[-5]
	-4	Y[-4]
	-3	Y[-3]
	-2	Y[-2]
Y[-1]	-1	Y[-1]
Y[0]	0	Y[0]
Y[1]	1	Y[1]
	2	Y[2]
	3	Y[3]
	4	Y[4]
	5	Y[5]
	6	Y[6]
	7	Y[7]
	8	Y[8]
	9	Y[9]
	10	Y[10]
	11	Y[11]
	12	Y[12]
	13	Y[13]
	14	Y[14]
	15	Y[15]
	16	Y[16]
	17	Y[17]
	18	Y[18]
	19	Y[19]
	20	Y[20]

C000000 Figure 4 The y-index in black follows the direction of the Y axis in the XY plane, from bottom to top. The gray-white y-index follows the Y-axis direction in the handwriting direction and the spreadsheet row count direction, from top to bottom.

This is a good example of how a simple and silly lack of a single universal standard convention leads to huge confusions in the interpretation of results in integer sequences.

8. C002112 Paraboctys.

Published at https://www.mersenneforum.org/showpost.php?p=634038&postcount=20.

The name "paraboctys" comes from one of the first reasonings from these studies in 2015/2016.

We first published paraboctys in three preprints in 2020. The paraboctys part nr.2 is at the link https://easychair.org/publications/preprint_open/7n9N.

8.1. Definition of a Paraboctys.

Paraboctys is a two-dimensional table. We define its formation law by $(d+1)$ integer sequences aligned (synchronized) in the horizontal rows.

If the sequences are polynomial, they are in X-axis direction in the form of $x = X[x]$ at $y = constant$.

Each set of $(d+1)$ elements aligned in the vertical column generates a polynomial sequence of degree d.

To do this, we use the equations that have been defined at https://easychair.org/publications/preprint_open/M18J.

The countless vertical columns form polynomials in the form of $y = Y[y]$ with degree d.

8.2. Notation for a Paraboctys.

The generic notation is:

$$PS\left[\text{equation}_1, \text{equation}_2, \text{equation}_3, \dots, \text{equation}_{(\text{d}+1)}\right]$$

Because of the standard of highest possible simplicity that we adopt at https://easychair.org/publications/preprint_open/M18J, then the generic notation to produce vertical polynomials is:

$$PS\left[X_{-flo\quad\left(\frac{d}{2}\right)}[x]@y = -floor\left(\frac{d}{2}\right); \dots; X_{-1}[x]@y = -1; X_0[x]@y = 0; X_1[x]@y = 1; \dots; X_{ceiling\left(\frac{d}{2}\right)}[x]@y = ceiling\left(\frac{d}{2}\right)\right]$$

For paraboctys with polynomial verticals in the form $y = Y0[y]$ of degree 0:

$$PS[X_0[x]@y = 0]$$

For paraboctys with polynomial verticals in the form $y = Y1[y]$ of degree 1:

$$PS[X_0[x]@y = 0; X_1[x]@y = 1]$$

For paraboctys with polynomial verticals in the form $y = Y2[y]$ of degree 2:

$$PS[X_{-1}[x]@y = -1; X_0[x]@y = 0; X_1[x]@y = 1]$$

For paraboctys with polynomial verticals in the form $y = Y3[y]$ of degree 3:

$$PS[X_{-1}[x]@y = -1; X_0[x]@y = 0; X_1[x]@y = 1; X_2[x]@y = 2]$$

For paraboctys with polynomial verticals in the form $y = Y4[y]$ of degree 4:

$$PS[X_{-2}[x]@y = -2; X_{-1}[x]@y = -1; X_0[x]@y = 0; X_1[x]@y = 1; X_2[x]@y = 2]$$

And so on.

8.3. Examples.

C002209 The oblong parabolic sieve of primes $PS[x + 2; x; x]$ at https://www.facebook.com/groups/snypo/posts/519692163021061/ and https://www.mersenneforum.org/showpost.php?p=616879&postcount=1.

C002210 The square parabolic sieve of primes $PS[x + 1, x, x + 1]$ at https://www.facebook.com/groups/snypo/posts/519692833020994/ and https://www.mersenneforum.org/showpost.php?p=616897&postcount=2.

8.4. Utility and use of Paraboctys.

We can use paraboctys to rotate increasing or decreasing the coefficient of highest degree of vertical polynomials. For each rotation, we can add positive or negative integers creating 3D bodies.

For example, we can view any 2nd degree polynomial as a parabola or a hyperbola. Only with 3D paraboctys can we see sequences of integers form all possible variations of Hyperbolic Paraboloid https://mathworld.wolfram.com/HyperbolicParaboloid.html. Of course, we can extend this to higher degrees of polynomials.

We can use paraboctys to explain how the GCs (composite generators) bound polynomial sequences of prime numbers (Bouniakowsky).

This helps us to see where all the infinite semiprimes formation (Goldbach) as well as the distribution of prime numbers over the two main GCs which are squares and oblongs (Legendre).

We understand the study of divisors of whole numbers with paraboctys.

We can only easily understand and see the prime numbers generated by the class of CCCP numbers https://www.mersenneforum.org/showthread.php?t=27328, because we will show that all divisors fill a paraboloidal and hyperbolic plane at the same time. This occurs on the lattice of the paraboctys $PS[x + 2; x; x]$.

9. Notation for index direction in any polynomial sequence.

Any polynomial integer sequence has two directions.

This is the reason any polynomial has two recurrence equations.

So, if the direction is:

$$Yd[y] \equiv \{\dots e, f, g, h, i, j, k \dots\} = \backslash\{\dots k, j, i, h, g, f, e \dots\}\backslash$$

Then, the reverse direction is:

$$\backslash Yd[y]\backslash \equiv \{\dots k, j, i, h, g, f, e \dots\} = \backslash\{\dots e, f, g, h, i, j, k \dots\}\backslash$$

In these studies, if an OEIS sequence is $Axxxxxx$, then we write the representation of that sequence in the reverse direction as $\backslash Axxxxxx\backslash$.

Example at C000663 https://www.facebook.com/groups/snypo/posts/290564229267190/

10. Notation for a negative polynomial sequence.

Any polynomial integer sequence has its negative.

If the positive is:

$$Yd[y] \equiv \{\ldots, e, f, g, h, i, j, k, \ldots\} = \backslash\{\ldots, k, j, i, h, g, f, e, \ldots\}\backslash$$

Then, the negative is:

$$-Yd[y] \equiv \{\ldots, -e, -f, -g, -h, -i, -j, -k, \ldots\} = \backslash\{\ldots, -k, -j, -i, -h, -g, -f, -e, \ldots\}\backslash$$

We consider all the columns of the paraboctys (or tables) as having the direction of the Y-axis and all the rows as having the direction of the X-axis.

In these studies, if an OEIS sequence is $Axxxxxx$, then we note the representation of that negative sequence as $-Axxxxxx$, or $-(Axxxxxx)$, or $(-Axxxxxx)$.

Reversing the direction or reversing the sign does not change our classification of the polynomial sequence.

Thus, every polynomial sequence has four forms:

- Positive.
- Negative.
- Positive reversed direction.
- Negative reversed direction.

Example at C000663.

11. Notation for interleave, union and joining sequences.

11.1. Notation for the interleave of sequences.

The interleave character is &.

Being the infinite polynomial sequence $Axxxxxx = Y_1[y] \equiv \{\ldots,1,f,3,h,5,j,\ldots\}$ and a second infinite polynomial sequence $Ayyyyyy = Y_2[y] \equiv \{\ldots 1,2,3,4,5,6 \ldots\}$, let us define the interleaving their elements while maintaining the duplicity of the elements.

The denotation is:

$$Axxxxxx\&Ayyyyyy = Y_1[y]\&Y_2[y] \equiv \{\ldots,1,1,f,2,3,3,h,4,5,5,j,6,\ldots\}$$
$$Ayyyyyy\&Axxxxxx = Y_2[y]\&Y_1[y] \equiv \{\ldots,1,1,2,f,3,3,4,h,5,5,6,j,\ldots\}$$

Sometimes there is no distinction between interleaving and union. Example at C000663 https://www.facebook.com/groups/snypo/posts/290564229267190/.

Example with distinction between interleaving and union: the interleave of the sequences https://oeis.org/A007590 and https://oeis.org/A000982 result in https://oeis.org/A166515 where we keep the repeated elements.

11.2. Notation for the union of sequences.

The union character is ∪.

Being the infinite polynomial sequence $Axxxxxx = Y_1[y] \equiv \{\ldots,1,f,3,h,5,j,\ldots\}$ and a second infinite polynomial sequence $Ayyyyyy = Y_2[y] \equiv \{\ldots 1,2,3,4,5,6 \ldots\}$, let us define the union of their elements while not maintaining the duplicity of the elements.

The denotation is:

$$AxxxxxxUAyyyyyy = Y_1[y] \cup Y_2[y] \equiv \{\ldots,1,f,2,3,h,4,5,j,6,\ldots\}$$
$$AyyyyyyUAxxxxxx = Y_2[y] \cup Y_1[y] \equiv \{\ldots,1,2,f,3,4,h,5,6,j,\ldots\}$$

Example: the union of the sequences https://oeis.org/A007590 and https://oeis.org/A000982 result in different sequence from https://oeis.org/A166515. Axxxxxx={0, 1, 2, 4, 5, 8, 12, 13, 18, 24, 25, 32, 40, 41, 50, 60, …). This new sequence does not have repeated elements.

Example for union or interleave resulting in the same sequence:

https://oeis.org/A052539 ∪ https://oeis.org/A087289 = https://oeis.org/A000051.

https://oeis.org/A052539 & https://oeis.org/A087289 = https://oeis.org/A000051.

(poderíamos diferenciar interleave de union da seguinte forma:

Interleave A com B: primeiro coloca as duas sequencias em fase, sempre no offset 0. Depois usa-se o primeiro elemento de A com index 1 para depois usar o elemento de B com index 1, e assim por diante. Ou seja, A interleave B resulta em uma sequência diferente de B interleave A. Mas em ambos os casos os elementos estarão em fase.

Union A com B não se preocupa em colocar as sequencias em fase e não se preocupa em repetir elementos comuns entre as sequencias.

Interleave se comporta como um zíper entre as duas sequencias. Union junta os elementos iguais e não se importa com a defasagem entre as sequencias.)

11.3. Notation for joining sequences.

The joining character is $+$.

Let us use this operation only to join the finite sequences and/or infinite sequences that have a first (or a last) element.

For example:

- Sequence 1 = https://oeis.org/A056737 = A056737 = {0, 1, 2, 0, 4, 1, 6, 2, 0, 3, 10, …}.
- Sequence 2 = number 0.
- Sequence 3 = https://oeis.org/A063655 = A063655 = {2, 3, 4, 4, 6, 5, 8, 6, 6, 7, 12, …}.

So, we denote the sequence {…, 12, 7, 6, 6, 8, 5, 6, 4, 4, 3, 2, 0, 0, 1, 2, 0, 4, 1, 6, 2, 0, 3, 10, …} as $\backslash A063655\backslash{+}0+A056737$.

12. Map of colors for all figures and tables.

12.1. The standard colors of the integer numbers:

https://oeis.org/A000004 The number 0, in red web color #FF0000.
https://oeis.org/A000012 The positive and negative number 1, in light-blue web color #00FFFF.
https://oeis.org/A000040 The positive and negative prime numbers, in blue web color #4F81BD.
https://oeis.org/A000290 The positive and negative square numbers (except 0 and ±1), in yellow web color #FFFF00.
https://oeis.org/A002378 The positive and negative oblong numbers (except 0 and ±2), in red-dark web color #963634.
https://oeis.org/A005563 The positive and negative (square–1) numbers (except 0, ±1, and ±3), in orange-dark web color #E26B0A.
https://oeis.org/A121893 All positive and negative composite numbers that are not a square number, or an oblong number, or a (square–1) number, in light-orange web color #FDE9D9.
CGs green lines in XY-plane, in dark-green web color #006400.

We will call the square, and oblong, and $(square-1)$ numbers as the SOM numbers.

12.2. The standard colors of the distinct and repeated composite numbers in TMT:

The web color #C4D79B represents the distinct composites with two pairs of complementary positive divisors in TMT.
The web color #76933C represents the repeated composites with two pairs of complementary positive divisors in TMT.
The web color #FDE9D9 represents the distinct composites with more than two pairs of complementary positive divisors in TMT.
The web color #DA9694 represents the repeated composites with more than two pairs of complementary positive divisors in TMT.

12.3. The standard colors of the positive and negative numbers:

The positive numbers are blue: fill= #DCE6F1 font= #000090.
The negative numbers are red: fill= #F2DCDB font= #FF0000.

12.4. The standard colors of the curves:

The yellow web color #FFFF00 diagonals are the SUB parabolas.
The Green web color #E2EFD9 diagonals are the ACC parabolas.
The Brown web color #943634 diagonals are the DES parabolas.
The Magenta web color #FF40FF is the SUB lines or square type curves. Even numbers.

The Violet web color #7F00FF is the DES lines or oblong type curves. Odd numbers.

12.5. The standard colors of the divisors:

For divisors $y < \sqrt{x}$: #60497a or RGB (96, 73, 122).

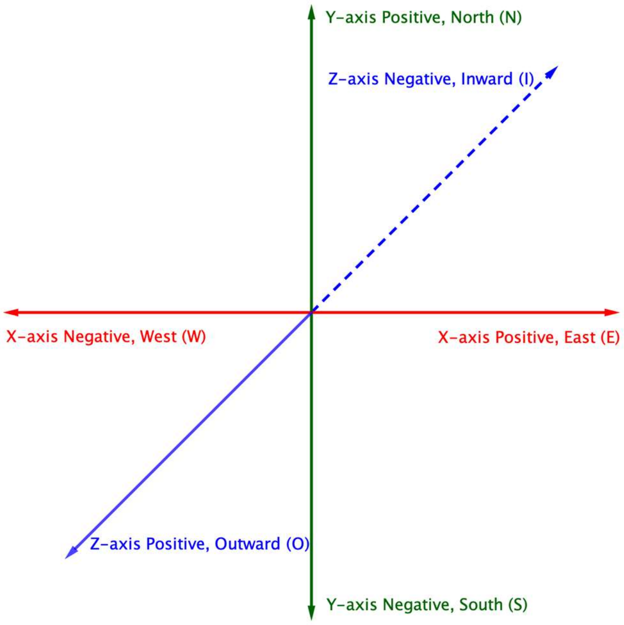
For divisors $y = \sqrt{x}$: #FF0000.

For divisors $y > \sqrt{x}$: #215967 or RGB (33, 89, 103).

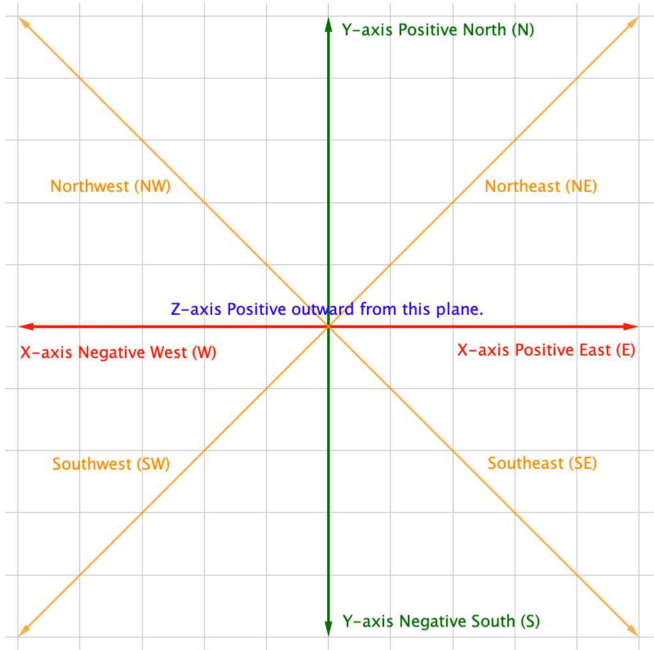
13. 3D orientation.

Following the GeoGebra convention:

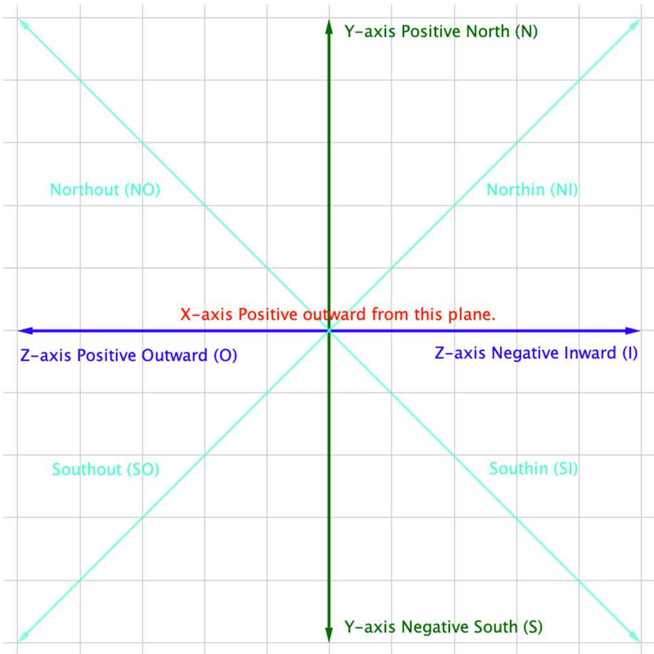
- X-axis in red
- Y-axis in green
- Z-axis in blue



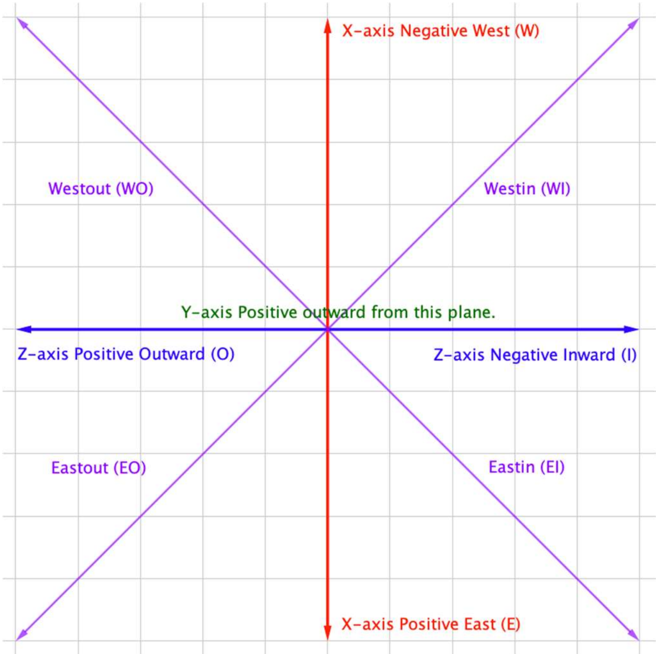
C000000 Figure 5 The 3D space and its XYZ axis.



C000000 Figure 6 The XY plane.



C000000 Figure 7 The YZ plane.



C000000 Figure 8 The ZX plane.

14. Submarine, Destroyer, and Aircraft-Carrier planes.

When defining inclined planes in Cartesian 3D space, they always are perpendicular to one of the three XY, YZ, or ZX planes and parallel to one of the three Z, X, or Y axes, respectively.

When drawing these inclined planes, the horizontal or the vertical axis of the inclined plane sometimes have the points of the Cartesian intersection and sometimes does not. This uncertainty is because it may occur exactly in the middle between two Cartesian intersections.

Thus, we define the SUB (SUBmarine) geometric plane as the inclined plane whose both horizontal and vertical axis has the points of the Cartesian intersection.

We define as DES (DEStroyer) geometric plane the inclined plane whose horizontal or vertical axis does not have the points of the Cartesian intersection because it is exactly in the middle between two Cartesian intersections. On the inclinations of 45˚ and 135˚, the axis that do not have the points of the Cartesian intersection is exactly in half between two Cartesian intersections.

We define as ACC (AirCraft-Carrier) geometric plane the inclined plane whose horizontal or vertical axis does not have the points of the Cartesian intersection because it is between two Cartesian intersections but not in exactly the middle. On the inclinations different of 45˚ and 135˚, the axis that does not have the points of the Cartesian intersection is not exactly halfway between two Cartesian intersections.

15. Symmetry Point (sp) in polynomials.

More info at https://www.mersenneforum.org/showthread.php?t=28269.

Notice that, if we want to have a symmetric polynomial, we just may do:

$$Y[y] = a_d y^d$$

This means that any Taylor shift in this curve results in:

$$Y[y+h] = a_d y^d + d a_d h y^{d-1} + \frac{d(d-1)}{2!} a_d h^2 y^{d-2} + \frac{d(d-1)(d-2)}{3!} a_d h^3 y^{d-3} + \cdots + \frac{d(d-1)(d-2)}{3!} a_d h^{d-3} y^3 + \frac{d(d-1)}{2!} a_d h^{d-2} y^2 + d a_d h^{d-1} y^1 + a_d h^d$$

The two expressions above represent the same sequence of integers but with an offset.

What changes is only the displacement along the Y-axis. Then, in each of the two curves there is a point of symmetry.

Because,

$$Y[y+h] = a_d (y+h)^d + a_{d-1}(y+h)^{d-1} + a_{d-2}(y+h)^{d-2} + \cdots + a_3 (y+h)^3 + a(y+h)^2 + b(y+h) + c$$

Or,

$$Y[y] = a_d y^d + a_{d-1} y^{d-1} + a_{d-2} y^{d-2} + \cdots + a_4 y^4 + a_3 y^3 + a y^2 + b y + c$$

Then, we will define the Y-coordinate of the symmetry point as being the value of y when:

$$Y^{[d-1]}[y] = \frac{d^{d-1}}{dy^{d-1}}(Y[y]) = 0$$

$$Y^{[d-1]}[y] = d!\, a_d y + (d-1)!\, a_{d-1}$$

And for $y = y_{sp}$:

$$d!\, a_d y_{sp} + (d-1)!\, a_{d-1} = 0$$

$$y_{sp} = -\frac{(d-1)!\, a_{d-1}}{d!\, a_d}$$

$$y_{sp} = -\frac{a_{d-1}}{d a_d}$$

And,

$$x_{sp} = Y[y_{sp}] = Y\left[-\frac{a_{d-1}}{d a_d}\right]$$

So,

$$symmetry\ point = sp = (x_{sp}, y_{sp}) = \left(Y\left[-\frac{a_{d-1}}{d a_d}\right], -\frac{a_{d-1}}{d a_d}\right)$$

When the polynomial generates a symmetric integer sequence, then the symmetry point coincides with one of the inflection points and/or with one of the Real roots.

When the polynomial generates an asymmetric integer sequence, then the point of symmetry will be closer to the most central inflection point of the curve and/or the most central Real root, if any.

16. Glossary of letters and abbreviations adopted.

Δ	The discriminant of a second-degree polynomial.
$¶(d)$	Product of divisors. https://oeis.org/A007955
$¶(proper)$	Product of proper divisors. https://oeis.org/A007956
$1dsr(x)$	First digit after decimal point of square root of x. C001171 https://www.facebook.com/groups/snypo/posts/515890850067859/ https://oeis.org/A023961.
$2dsr(x)$	Second digit after decimal point of square root of x. C001172 Sequence https://oeis.org/A111862.
$3dsr(x)$	Third digit after decimal point of square root of x. C001173 Sequence https://oeis.org/A328819.
$4dsr(x)$	Fourth digit after decimal point of square root of x. C001174 Sequence https://oeis.org/A328820.
a	Coefficient of the second-degree term of a polynomial. Also, $a = a_2$.
a_n	Coefficient of the n-degree term of a polynomial.
$a_n{}^{º}$	Coefficient of the n-degree term of a polynomial in offset $f = 0$.
$A - parabola$	Parabola in the form of $y = -\lvert a\rvert x^2 + bx + c$. Because $-\lvert a\rvert < 0$, then the aperture of the parabola is facing down. This reminds us of the letter “A”. Parabola A type.
ACC	Abbreviation for aircraft-carrier or ACC-type polynomial. It means that there are no two elements equidistant to the symmetry point of the polynomial. Neither is there an element that is the symmetry point of the polynomial. See https://www.mersenneforum.org/showpost.php?p=633531&postcount=6.
$ACC - plane$	Aircraft-carrier plane.
$a^{º}$	Coefficient of the second-degree term of a polynomial in offset $f = 0$.
b	Coefficient of the first-degree term of a polynomial. Also $b = a_1$. The distance between the two index zeroes of the GCs in the form of $CG[y] = y^2 + by = y(y \pm b)$. The coefficient of the quadratic equation $x = ay^2 + by + c$. The sum of the roots is $-b/a$. The symmetry point is $-b/2a$. The coefficient of the hyperbola $xy = bx + cy$.
$b2$	The difference between the complementary divisor of $d2$ and the divisor $d2$.

	Being $x = d2 * d´2$, we will denote it as: $$b2 = d´2 - d2 = \frac{x}{d2} - d2$$
$b3$	The difference between the complementary divisor of $d3$ and the divisor $d3$. Being $x = d3 * d´3$, we will denote it as: $$b3 = d´3 - d3 = \frac{x}{d3} - d3$$
$b_d = d_2 - d_1$	The difference between the divisors of the pairs of complementary divisors $b_d = d_2 - d_1$ of an integer $x = d_1 * d_2$. C000246 https://www.facebook.com/groups/snypo1/posts/1906256426217935. The b_d coefficient of the hyperbola $xy = b_d x + cy$ at the MID.
$b_{dm} = d_{c2} - d_{c1}$	The smallest (**m**inimum) difference between the two complementary positive divisors of all pairs of complementary divisors of an integer x. It is the difference between the central pair of complementary divisors of an integer $x = d_{c1} * d_{c2}$: $min[b_d] = b_m = b_{dm} = d_{c2} - d_{c1}$. The difference between the divisors of the central pair of complementary divisors is: b_{dm} =https://oeis.org/A033677[x] −https://oeis.org/A033676[x] = https://oeis.org/A056737
$b_{dM} = d_{t2} - d_{t1} = x - 1$	The largest (Maximum) difference between the two complementary divisors of all pairs of complementary divisors of an integer x. It is the difference of the trivial pair of complementary divisors of an integer $x = d_{t1} * d_{t2}$: $$max[b_d] = b_M = b_{dM} = d_{t2} - d_{t1} = x - 1$$
$b_q = d_r + d_s$	The sum of the two complementary divisors $b_r = d_r + d_s$. AXXXXXX
$b_r = d_r - d_s$	The difference between the two complementary divisors $b_r = d_r - d_s$. C000422 https://www.facebook.com/groups/snypo/posts/341164967540449/ = https://oeis.org/A350576.
$b_s = d_2 + d_1$	The sum of the two complementary divisors $b_s = d_2 + d_1$ of an integer $x = d_1 * d_2$. The b coefficient of the parabola $x = y^2 + by + c$ at the MID.
$b_{sm} = d_{c2} + d_{c1}$	The smallest (minimum) sum of the divisors among the pairs of complementary divisors of an integer x. It is the sum of the central pair of complementary divisors of an integer $x = d_{c1} * d_{c2}$: $$min[b_s] = b_{sm} = d_{c2} + d_{c1}$$ The sum of the central pair of complementary divisors is: b_{sm} =https://oeis.org/A033676[n] +https://oeis.org/A033677[n] =https://oeis.org/A0636555
$b_{sM} = d_{t2} + d_{t1} = x + 1$	The largest (Maximum) sum of the divisors among the pairs of complementary divisors of an integer x. It is the sum of the trivial pair of complementary divisors of an integer $x = d_{t1} * d_{t2}$:

	$max[b_s] = b_{sM} = d_{t2} + d_{t1} = x + 1$
bot	Abbreviation for the bottom.
c	Constant term of a polynomial. Also, $c = a_0$. When capitalized, C is the table column. If no other note, when referenced in our tables, the c or C coefficient always appears along the X-axis, and $x = C$. The constant coefficient of the quadratic equation $x = ay^2 + by + c$. In quadratics, the product of the roots is c/a. The coefficient of the hyperbola equation $xy = bx + cy$. In hyperbola, c is the product between the symmetrical terms divided by the transverse axis.
$Cartesian\ plane$	It is the simple square Lattice Grid. No element in the plane has a specific value. It is just a grid.
$CG\ or,$ $CGs\ (plural)$	Composite Generator. All the polynomials with at least one element Zero. General equation: $CG[y] = y(Y[y])$, where $Y[y] = polynomial\ any\ degree$. Example of quadratic CG: https://www.mersenneforum.org/showthread.php?t=27420&page=2
c^{o}	Constant term of a polynomial in offset $f = 0$.
$C - parabola$	Parabola in the form of $x = \|a\|y^2 + by + c$. Because $\|a\| > 0$, then the aperture of the parabola is facing right. This reminds us of the letter “C”. Parabola C type.
$CompositePi[x]$	The number of composite numbers $\leq x$.
$d3$	The third smallest positive divisor of a non-negative integer. If the integer is a composite number, then its third divisor is either a prime number or a square number. https://oeis.org/A340768.
$d'1$	The complementary divisor of $d1$ such as $1 = d1 * d'1 = 1 * 1$.
$d'2$	The complementary divisor of $d2$ such as $x = d2 * d'2$.
$d'3$	The complementary divisor of $d3$ such as $x = d3 * d'3$.
d_1	The smallest complementary divisor $d_1 \leq \pm\sqrt{\|x\|} \leq d_2$, and $x = d_1 * d_2$.
d_2	The largest complementary divisor $d_2 \geq \pm\sqrt{\|x\|} \geq d_1$, and $x = d_1 * d_2$.
$(d_1; d_2)$	The pair of complementary divisors such that $d_2 \geq \pm\sqrt{\|x\|} \geq d_1$, $x = d_1 * d_2$. Because of the symmetry in the multiplication table, we will always consider it to be just a single pair of complementary divisors when we exchange the sign of the two divisors of the pair. $(d_1; d_2) = (-d_2; -d_1)$.
d_{c2}	The largest central complementary divisor $d_{c2} \geq \pm\sqrt{\|x\|} \geq d_{c1}$, and $x = d_{c1} * d_{c2}$. For positive divisors, d_{c2} is the sequence https://oeis.org/A033677.

$(d_{p1}; d_{p2})$	The proper pair of complementary divisors such that $x > d_{p2} \geq +\sqrt{\lvert x \rvert} \geq d_{p1} \geq 1$ and $d_{p1} * d_{p2} = x = positive\ integer$.
d_p	Reserved for proper divisors reference $1 \leq d_p < x$.
d_{p1}	The smallest complementary proper divisor $1 \leq d_{p1} \leq +\sqrt{\lvert x \rvert}$. If $d_{p1} = 1$, there is no d_{p2}, because $d_{p2} \neq x$.
d_{p2}	The largest complementary proper divisor $1 < +\sqrt{\lvert x \rvert} \leq d_{p2} < x$.
d_r	The complementary divisor of d_s. The integer is $x = d_s * d_r$. It is the sequence https://oeis.org/A350509.
d_s	The number of divisors of the first consecutive sequence of divisors of an integer n. It is the sequence https://oeis.org/A055874. The complementary divisor of d_r.
$(d_{t1}; d_{t2})$	The trivial pair of complementary divisors such that $d_{t2} \geq d_{t1}$ and $d_{t1} * d_{t2} = x = integer$. Then, because $d_{t1} \leq 1$, then d_{t1} can be $-\lvert x \rvert$, -1, or 1. Because $d_{t2} \geq -1$, d_{t2} can be -1, 1, or $\lvert x \rvert$.
d_t	Reserved for the trivial pair of complementary divisors reference.
d_{t1}	The smallest trivial complementary divisor. $d_{t1} \leq \pm\sqrt{\lvert x \rvert} \leq 1 < d_{t2}$. Can be $-\lvert x \rvert$, -1, or 1.
d_{t2}	The largest trivial complementary divisor. $d_{t2} \geq \pm\sqrt{\lvert x \rvert} \geq -1 > d_{t1}$. Can be -1, 1, or $\lvert x \rvert$.
DES	Abbreviation for destroyer or DES-type polynomial. The symmetry point is not an element of the polynomial. The symmetry point is equidistant from all elements in the pairs of duplicated elements of the polynomial. See https://www.mersenneforum.org/showpost.php?p=633531&postcount=6.
$DES - plane$	A DES-type plane.
D_n	The absolute value of the n-th divisor of an integer. Always $Dn > 0$.
$D - parabola$	Parabola in the form of $x = -\lvert a \rvert y^2 + by + c$. Because $-\lvert a \rvert < 0$, then the aperture of the parabola is facing left. This reminds us of the letter "D". Parabola D type.
e	The eccentricity of hyperbola.
$EvenCompositePi$	The number of even composite numbers $\leq x$.
$EvenPi[x]$	The number of even numbers $\leq x$.
$EvenPrimePi[x]$	The number of even prime numbers $\leq x$.
f	The offset value. $f = integer$.
h	The Taylor shift value. $h = Real\ number$.
HL	HL is the short name for the Hyperbolic Lattice grid. HL is the base to construct FMT. The function is $HL[x, y] = xy$.
$HPSP\ diagonal$	A line formed by any two distinct HPSP points of the PSP parabola.

$HPSP\ point$	Any point over PSP parabola crossing a hyperbola in the form of $xy = bx + cy$.
$HSP.HL$	The "Hyperbolic Sieve of Primes in HL - Hyperbolic Lattice grid". C000433 https://www.facebook.com/groups/snypo/posts/357128065944139/.
$HSP.MHL$	The "Hyperbolic Sieve of Primes in MHL – Modular Hyperbolic Lattice grid".
HS	Hyperboctys.
$L[x, y, z]$	3-dimension (3D) lattice grid.
$L[x, y]$	2-dimension (2D) lattice grid.
$lpp[x]$	Largest prime power $\leq x$. https://oeis.org/A031218.
LR	Latus Rectum of a conic line.
$M[x]$	Exponential of Mangoldt function. https://oeis.org/A014963.
MT	Multiplication Table.
MHL $x\ mhl\ y$	The Modulo Hyperbolic Lattice Grid. The general equation is: $MHL[x, y > 0] = x\ mhl\ \lvert y\rvert = 1 + (x - 1\ (mod\ \lvert y\rvert))$ $MHL[x, y < 0] = x\ mhl\ (-\lvert y\rvert) = -1 + (x + 1\ (mod\ (-\lvert y\rvert)))$ MHL is the base to construct MID.
MID	Map of the integers and their divisors. It is based on MHL. OEIS sequences related to MID: https://oeis.org/A027750 and https://oeis.org/A350380.
n	An integer number.
$NoCG$	Number of odd CG lines in PSO, or the number of pairs of complementary odd divisors minus 1 ref. trivial pair of complementary divisors.
$O1, O2, O3, O4, O5,$	When we divide the XY plane into eight equal parts, each part is an octant. The X and Y axes, plus two diagonals that cross each other forming eight 45° angles divide the plane into octants.
$Ob, or\ ob$	Oblong abbreviation.
$Oblong\ parabola$	We call an "oblong parabola" all the parabolas in the form of (oblong2 minus oblong1) regardless of offset. https://www.facebook.com/groups/snypo/posts/659997235657219/.
$OddCompositePi[$	The number of odd composite numbers $\leq x$.
$OddPi[x]$	The number of odd numbers $\leq x$.
$OddPrimePi[x]$	The number of odd prime numbers $\leq x$.
p^k	Powers of primes. https://oeis.org/A000961.
$PFp[Yd[y]]$	Power-free function.
prd	Prime root divisors. https://oeis.org/A350380 = M[d] = https://oeis.org/A014963 [https://oeis.org/A027750]
$PrimePi[x]$	The number of prime numbers $\leq x$.
PS	Paraboctys for quadratics or polyboctys for distinct orders.
PSO	The "Parabolic Sieve of Primes with Offset".
PSP	The "Parabolic Sieve of Primes". Matiyasevich, Yuri and Stechkin, Boris. (1999) "A visual Sieve for Prime Numbers", available online at:

	https://logic.pdmi.ras.ru/~yumat/personaljournal/sieve/sieve.html
$PSP\ parabola$	PSP parabola in MID are the two parabolas: $x = \pm y^2$.
$Q1, Q2, Q3, Q4$	The X and Y axes divide the plane into four quadrants, which cross each other, forming the four 90° angles.
$R1, R2$	The two roots of a quadratic equation.
RIT	RIT is the short form for the Right-Isosceles Triangle in HL - Hyperbolic Lattice grid.
$roundz[x]$	New function called "round half to zero". Isomorphic with the round function, except when rounding $(integer \pm 0.5)$. For $m = integer$ the roundz function is: $$roundz[m + 0.5] = m$$ $$roundz[m - 0.5] = m - 1$$ Or $$roundz\left[\frac{2k+1}{2}\right] = k$$ $$roundz\left[\frac{2k-1}{2}\right] = k - 1$$
s	Complex number. Real and imaginary parts are the function $s = X[x] + iY[y]$.
SL	Circular Square Lattice Grid.
SMT	Square Multiplication Table. Only the positive products of FMT.
$SNYPO$	See paragraph 2.1 above.
SOM	We will call the square, oblong, and $(square - 1)$ numbers the demarcation numbers. This is because the only numbers that are common to other classifications are the numbers 0, 1, 2, and 3. See paragraph 3 above.
$sopfr[x]$	Sum of prime factors repeated. https://oeis.org/A001414.
sp	The symmetry point of a polynomial curve. See the definition at https://www.mersenneforum.org/showthread.php?t=28269.
$sp_{Yd[y]}$	The symmetry point of a polynomial with degree d and index y.
$spp[x]$	Smallest prime power $\geq x$. https://oeis.org/A000015.
$Sq\ or\ sq$	Square abbreviation.
$Square\ lattice$	The simple square lattice grid is the Cartesian plane. No element in the plane has a specific value. It is just a grid. See SL and OL.
$Square\ parabola$	We call a "square parabola" all the parabolas in the form of (square2 minus square1) regardless of offset. https://www.facebook.com/groups/snypo/posts/659997058990570/.

SUB	Abbreviation for submarine or SUB-type polynomial. It means there is one point or one element that is the symmetry point of the polynomial. All other pairs of duplicated elements are equidistant to the symmetry point. See https://www.mersenneforum.org/showpost.php?p=633531&postcount=6.
$SUB - plane$	SUB-type plane. In this case, there is a symmetry line instead of a symmetry point.
TMT	The short name for Triangular Multiplication Table. Sub-set of FMT.
TN	The short name for Triangle of Naturals.
TSP	The short name for Triangular Sieve of Primes.
$TSP\ diagonal$	Triangular Sieve of Primes diagonals or TSP diagonals are any HPSP diagonal intersecting the X-axis and Y-axis integer coordinates.
TZ	The short name for Trianz.
$U - parabola$	Parabola in the form of $y = \|a\|x^2 + bx + c$. Because $\|a\| > 0$, then the aperture of the parabola is facing up. This reminds us of the letter "U". Parabola U type.
$X[x]$	Polynomial function X with index x. This is a 1-dimension lattice grid.
x_1, x_2, x_3	Three constant and consecutive elements form a parabola.
x_{focus}	x-coordinate of the focus of a parabola.
x_{sp}	x-coordinate of the symmetry point.
$Xd[y]$	Polynomial function X with degree d and index x.
y	The index of an integer sequence. It follows the Y-axis direction.
$Y[y]$	Polynomial function Y with index y. This is a 1-dimension lattice grid.
y_{sp}	y-coordinate of the symmetry point.
$YB\ diagonal$	Yuri-Boris' diagonals or YB diagonals in MID are all the lines discovered by Yuri Matiyasevich and Boris Stechkin in "A visual Sieve for Prime Numbers" at https://logic.pdmi.ras.ru/~yumat/personaljournal/sieve/sieve.html.
$Yd[y]$	Polynomial function Y with degree d and index y.

Acknowledgments.

I would like to thank all the essential support and inspiration provided by Mr. H. Bli Shem and my Family. A Talmud proverb says, “He who does not teach his son a trade teaches him to steal”. The wisest of trades is study. I dedicate these studies to my children.

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