Probing Nano-Hz Gravitational Waves of Axion/Axion-like Particles

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The most conventional approaches to find axion-like particles (ALP) or notably axions typically lies on their coupling with photons. However, if the coupling is extremely weak then there is a chance that they decouple themselves from the standard model physics and becomes invisible which are a source of stochastic gravitational waves at the epoch of the early universe formation, solving the riddle of the axion origin mystery and the origin of weak gravitational waves. Having the axion decay constant rates as $f \geq 10^{16^{+1}_{-1}}\mathrm{GeV}$, the axion signals which can be detected by either ground/space based observatories or pulsar timing arrays shows a broad space parameter of axion mass thus helping to probe the existents of invisible axions originating in the early universe. The ALP or axions generally couple to a dark gauge bosons which at the onset of oscillations produces tachyonic instabilities that increases the visible parameter for the ALP or axion dark matters. The quantum fluctuations that arises and getting amplified by the strong coupling of axion/ALP to dark boson modes sources chiral gravitational waves (GWs). The accurate spectrum of these GWs have been calculated from the U(1) gauge fields produced by axion dark matters. The explosive outbursts of gauge fields indicates the advantage of non-linear data analysis over linear modes to calculate the exact GWs spectrums. The ground/space based interferometers and pulsar timing arrays have the ability to probe the bottom up approach of the axions, in the weakly coupled regime which otherwise remains unconstrained. Further, it has been discussed the kinetic mixing mechanism and the dark gauge photon mass over the insensitivity of the couplings to standard model fields. The ALP scenarios or realistic axions may provide us useful informations about the signal templates of the early universe, as well as useful datas for GW experiments. Throughout the paper we will assume the axion field being homogeneous the equations of motions for the gauge boson modes depend on the parametric valued scales $|\overrightarrow{k}| = k$.

I. Introduction –

Apart from the baryonic matter, cosmological observations hinted a specially different types of matter called the dark matter, that takes up a major portion of the universe. In the preceding years where the direct detection of GWs by space/ground based observatory, pulsar timing arrays opened up new avenues to explore the phenomenology's of the early universe. And, in particular, the source of the GWs from the axions or ALP dark matters (DMs) [1, 2, 3] probed the way for developing an extension to the standard model physics e.g, in the fundamentals of the string theory [4], probing the DM candidates [4, 5, 6], providing a suitable cosmological problem for inflation [7], the solution to electroweak hierarchy problem [8], or to solve the strong charge-parity (CP) problem [9] which is a symmetry phenomenology for which the axions have been discovered via the Peccei-Quinn mechanism [10] or spontaneously broken symmetry at a high scales producing very light Nambu-Goldstone bosons. Black hole superradiance [11] gives us the hint of a bounded state, if the axions/ALPs decoupled from the standard model. The viable parametric space that has to be needed to probe the axions/ALP spans hugely, which makes them a challenging task, and also spans new avenues of freedom through innovative approaches and experiments. Several searches have been made through the indication of axion-photon coupling, that is a proportionality inverse to the axion decay constant $f{\sim}10^{16}$ which is a region of larger couplings (small decay rates) which is more constrained, however, the more large the value gets, the more difficult it is to probe [3].

The axion/ALP may leave a trace in the early universe in the form of stochastic gravitational wave background (SGWB) where they are found to couple with a light dark photons [2]. When the axion fields start to oscillate in the early universe, then the rolling gradient fields produces a tachyonic instability, that in general induces or amplifies the vacuum fluctuations which induces a time dependent anisotropic stress in the energy-momentum tensor with a resultant generation of SGWB. In such a process, At the initial phase of the axion oscillation, they tend to have a non-zero misalignment which when gets compared to Hubble parameters, then the behaviors' becomes same as DM candidates. a large energy stored in the axions/ALP transferred in the form of GWs [2, 12]. Moreover, it has been shown [13], the PQ-phase transition which when is strongly in the first order, then there are high probabilities for the production of GWs. The dependence of this model leads to a highly affirmative conclusions like the durationand nucleation temperature, on the PQ-symmetry breaking and QCD-gauge coupling for the production of GWs spectra.

Due to the nature of the small couplings with the SM particles, the mass of the hidden sector particles are much lighter than the electroweak scale. The concepts of vector portals/axion portals [14] helped us in proper understanding of those dark particles [15] and how they interact with the baryons. Known porals are [14] (i) the vector portal, $B_{\mu\nu}Z'^{\mu\nu}$ (ii) axion portal, $\left(\frac{a}{f_a}\right)F_{\mu\nu}\tilde{F}^{\mu\nu}$, … , (iii) the Higgs portal, $|S|^2H^\dagger H$, … , and (iii) the neutrino portal, LHN [16]. The dark matter can be a portal particle or could get coupled weakly to a poral particle through a hidden interaction like, it can couple to a vector boson [17] or an ALP [18]. According to the phenomenology, there can be the existence of the axion portal and higgs portal at the same time (rather coexistent) where a Higgs singlet can provide mass to a vector boson which can be a dark photon [14]. This suggests a rarely occurring Higgs decay, $H \rightsquigarrow Z'Z' \rightsquigarrow four-lepton\ state$ where the coexistence occurs in the way that, the first decay occurs through Higgs portal while the second decay occurs through vector portal [14, 19]. The mixing of the DM with the Higgs is very important as to the extent the DMs are considered in the context of modified gravity-dynamics like scalar-tensor models, $f(r)$ models, that if a graviton obtains mass by interaction with the DM particles then it can itself reflect as a DM candidate in the scheme of massive gravitons which are difficult to observe [20]. Therefore, if a graviton obtains a mass, then it can be a DM particle and this is established in [20] via a scalar chameleon field ϕ_c of nonminimal coupling [21, 22, 23, 24, 25, 26, 27].

The paper has been prepared under the following sections, Section II, discussing in detail the PQ phase transition of the first order and how gravitational waves are produced from them with the help of derived Coleman-Weinberg mechanism, then Section III, which discusses how gauge fields are produced from a massless $U(1)_X$ gauge field from the dark sector, with dark photon fields X_μ and the axion field potential ϕ_A. Then Section IV, the production of gauge fields with the Fourier modes that splits it into two polarization modes with the circular polarization modes being most suitable for tachyonic instabilities via the kinetic mixing mechanisms. Section V, the gravitational wave spectrums and equations describing them. Section VI, the gravitational wave detections and analysis of the H1-H2 detectors co-alignment either together or separate with the analysis of the S4 and S5 run.

II. Peccei-Quinn phase transition and gravitational waves –

To investigate the stable, classically scale-invariant axion/ALP sectors, with further mass parameters, it is indeed necessary to pave the model quantum mechanically rather than the classical Lagrangian [13] where the parameter space for the axion model could be taken from a TAF axion model [28] where the P-Q symmetry is broken quantum mechanically via Coleman-Weinberg (CW) mechanism [29] to generate the mass scales, perturbatively via quantum corrections, allowing a fully calculable setup [13].

In order to investigate the one-loop phase transition of the P-Q transition, we consider the effective potential [13, 30, 31],

$$\begin{aligned} V_{\mathrm{eff}}(\phi,T) \equiv \quad & V_{\mathrm{CW}}(\phi) + \Lambda_0 \\ & + \frac{\mathrm{T}^4}{2\pi^2}\left(\sum_b n_b\left(\int_0^\infty dq\ q^2\ \mathrm{In}\Big[1 \mp \exp\left(-\sqrt{q^2+x}\right)\Big]\left(\frac{M_b^2(\phi)}{T^2}\right)\right)\right. \\ & \left. - \sum_f n_f\left(\int_0^\infty dq\ q^2\ \mathrm{In}\Big[1 \mp \exp\left(-\sqrt{q^2+x}\right)\Big]\left(\frac{M_f^2(\phi)}{T^2}\right)\right)\right) \end{aligned} \tag{1}$$

Where, in $V_{\mathrm{eff}}(\phi,T)$, a constant term Λ_0 has been provided to account for the minimum potential ϕ of the cosmological constant Λ where the thermal correction term $V(T)$ has been evaluated in the 3rd order expansion to scale for the renormalization scheme in V_{CW}.

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V_{eff} can be considered as the function of ϕ for the critical temperature T_c and the $T=0$. For the effective potential $\langle\phi\rangle = 0$, $T > T_c$ which represents a false vacuum, but the true vacuum can only be considered when $\phi = \langle\phi\rangle \neq 0$ [13] and represented by the formalism [13, 32, 33, 34, 35],

$$\Gamma \approx \max\left(T^4 \frac{\left(\left(\frac{2\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)}\right)_3 \left(\int_0^\infty dr\, r^{d-1}\left(\frac{1}{2}\phi'^2 + V_{\text{eff}}(\phi,T)\right)\right)_3\right)}{2\pi T}\right)^{\frac{3}{2}} \exp\left(\frac{-\left(\frac{2\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)}\right)_3 \left(\int_0^\infty dr\, r^{d-1}\left(\frac{1}{2}\phi'^2 + V_{\text{eff}}(\phi,T)\right)\right)_3}{T}\right), \frac{1}{R_4^4}\left(\frac{\left(\frac{2\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)}\right)_4 \left(\int_0^\infty dr\, r^{d-1}\left(\frac{1}{2}\phi'^2 + V_{\text{eff}}(\phi,T)\right)\right)_4}{2\pi}\right)^2 \exp\left(-\left(\frac{2\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)}\right)_4 \left(\int_0^\infty dr\, r^{d-1}\left(\frac{1}{2}\phi'^2 + V_{\text{eff}}(\phi,T)\right)\right)_4\right) \tag{2}$$

Where $base\ 3$ and $base\ 4$ represents the action S_3 and S_4, where the average action is taken as S_d evaluated at O_d bounce with R_4 the size of the O_4 bounce above giving the scaling arguments as [36],

$$S_d = \frac{4\pi^{d/2}/\Gamma(d/2)}{(2-d)} \int_0^\infty dr\, r^{d-1} V_{\text{eff}}(\phi,T) \tag{3}$$

Where it has been checked in [13] that $S_3/T < S_4$, Γ is dominated by O_3 bounce which shows that the phase transition is typically due to thermal effects rather than quantum effects.

As for the other models of CW symmetry breaking [13, 37, 38] when $T < T_c$ scalar field ϕ is trapped under a false vacuum at $\phi = 0$ until the condition when $T \ll T_c$ and then the universe gets super cooled, the vacuum energy density dominates that tends to grow exponentially during inflation, with the Hubble rate $\widehat{H} = \sqrt{\beta} f_2^a \left(\frac{4}{\sqrt{3}} \widetilde{P_m}\right)$ where $\widetilde{P_m}$ is the reduced Plank's mass, the universe starts expanding until T reaches the Nucleation temperature T_n which corresponds to the temperature when $\Gamma/\widehat{H}^4 \sim 1$ dominated by the O_3 bounce, given the value of T_n as [13],

$$T_n = \frac{S_3 - 4\text{In}\left(T_n/\widehat{H}\right)}{\frac{3}{2}\text{In}\left(\frac{S_3}{T_n}\Big/2\pi\right)} \tag{4}$$

Now, when the value becomes, $T \ll T_n$ the GWs are produced due to the dominant source of the bubble collisions in the vacuum space-time. Due to the turbulence in the inflammatory behavior of the matter and radiation, the GWs becomes dominant at that early phase of universe. Where the peak of the red-shifted frequency that we observe today as [13, 39],

$$f_{peak} \approx 3.79 \times 10^2 \frac{\beta}{T_{RH}} \frac{T_{RH}}{10^{10}\text{GeV}} \left\{g^* \frac{T_{RH}}{100}\right\}^{1/6} \text{Hz} \tag{5}$$

Where T_{RH} is the re-heating temperature and $g^* = 2 \times 10^2$: the reasonable value given in SM to satisfy the TAF requirement [28].

III. Axions/ALP production from gauge fields –

Here we will use the methodology as described in [1]. Let's, consider a massless $U(1)_X$ gauge field from the dark sector, as dark photon fields X_μ and the axion field ϕ_A . the corresponding Lagrangian density is given by,

$$\mathcal{L}^{gauge} = -\frac{1}{2}\partial^\mu \phi_A \partial_\mu \phi_A - V(\phi_A) - \frac{1}{4} X^{\mu\nu} X_{\mu\nu} - \frac{\alpha}{4f}\phi_A X_{\mu\nu}\tilde{X}^{\mu\nu} \tag{6}$$

Where $X^{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu$ the field strength tensor, $\tilde{X}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} X_{\rho\sigma}/2\sqrt{-g}$ with g being the determinant of the metric, α be the dimensionless coupling constant and f the decay rate constant of the axion having a value of $\sim 10^{16}$. The potential is given by [1],

$$V(\phi_A) = \Lambda^4\left[1 - \cos\left(\frac{\phi_A}{f}\right)\right] \tag{7}$$

Where Λ is scale dynamics and that relates the axion mass m_A as $m_A = \Lambda^2/f$. Adopting the metric convention mechanism as [2] $ds^2 = a(\tau)^2\left(d\tau^2 - \delta_{ij} l^i l^j\right)$, ϕ_A being homogeneous, the space-time scale factor $a(t) \propto \sqrt{t}$, the equations for motion of the axion fields is given by [1],

$$\ddot{\phi}_A + 3H\dot{\phi}_A - \frac{1}{a^2}\nabla^2\phi_A + \frac{\partial V}{\partial \phi_A} = \frac{a}{4f} X_{\mu\nu}\tilde{X}^{\mu\nu} \tag{8}$$

With the Hubble parameter $H = \frac{\dot{a}}{a}$ with the axion rolls to the minimal coupling $\phi_A X_{\mu\nu}\tilde{X}^{\mu\nu}$ leading to the formation of the dark photon quanta fields X_μ . In the radiation dominated universe, when the Hubble expansion rate $H = T^2/\widetilde{P_m}$ where $\widetilde{P_m}$ is the reduced Planks mass, the axions oscillates and a tachyonic instability is formed, where the frequency of the gauge bosons grow exponentially, with a large amplification of the dark photon field X_μ, the excess energy being transferred to the dark radiation and due to this instability and large amplifications [2, 3] the GWs are produced.

IV. Dark photon and kinetic mixing –

As shown in [1] the axion can interact with the SM and in the process, amplify gauge fields, but, the SM photon which interacts via portals could acquire a thermal effective action which would make a radiation field universe. Hence, its better to assume, the gauge fields as hidden photons whose coupling is not kinematically prohibited and not thermalized during the phase of axion oscillation leading to a tachyonic instability mechanism. To develop a thermal mass [3], the m_X can be made to zero, where which is not physical as the thermal radiation dominates with the field strength $A = A'\sqrt{1-\epsilon^2}$, however what is physical is the induced thermal mass $\Psi_{m(T)}$ could be redefined with the relation $m_X \neq 0$ as in the dispersion relation [3],

$$\left[\omega^2 + k^2 + \begin{pmatrix} \epsilon^2 {m_X}^2 + \Psi_{m(T)} & -\epsilon' {m_X}^2 \\ -\epsilon' {m_X}^2 & {m_X}^2 \end{pmatrix}\right]\begin{pmatrix} A'^\mu \\ X'^\mu \end{pmatrix} = 0 \tag{9}$$

Where $\epsilon' = \frac{\epsilon}{\sqrt{1-\epsilon^2}}$ with the photon thermal mass $\Psi_{m(T)} = e^2T^2$ and the oscillation evaluation of the axion is given by $\Psi_{m(T)}{}^{OSC} = e^2 m\widetilde{P_m}$.

Following the Ref. [2, 1, 3, 40, 41, 42, 43], to study the production of the dark photons, the gauge fields can be quantized by,

$$\hat{X}^i(\boldsymbol{\varsigma},\tau) = \int \frac{d^3k}{(2\pi)^3}\hat{X}^i(\boldsymbol{k},\tau)e^{i\boldsymbol{k}\cdot\boldsymbol{\varsigma}} = \sum_{\lambda=\pm} \frac{d^3k}{(2\pi)^3} v_\lambda(k,\tau)\varepsilon_\lambda^i(\boldsymbol{k})\hat{a}_\lambda(\boldsymbol{k})e^{i\boldsymbol{k}\cdot\boldsymbol{\varsigma}} + h.c. \tag{10}$$

Where, to see the exponential growth, the Fourier modes of the gauge field splits it into two polarization modes as [1],

$$\begin{gathered} \boldsymbol{A}(\boldsymbol{k},\tau) = A_+(k,\tau)e^+\boldsymbol{k} + A_-(k,\tau)e^-\boldsymbol{k} \\ \widehat{\boldsymbol{k}}\cdot e^\pm = 0 \\ i\widehat{\boldsymbol{k}}\cdot e^\pm = 0 \\ \widehat{\boldsymbol{k}} \equiv \boldsymbol{k}/|\boldsymbol{k}| \end{gathered} \tag{11}$$

Thus, determining the equations of motion with the Hubble rate and circular polarization modes as [1],

$$\ddot{A}_\pm + H\dot{A}_\pm + \left(\frac{k^2}{a^2} \mp \frac{k}{a}\frac{a\dot{\phi}_A}{f}\right)A_\pm = 0 \tag{12}$$

Showing that depending on the sign of the potential $\dot{\phi}_A$, one of the mode is tachyonic leading to instability and exponential amplification of the gauge amplitude. When the $\dot{\phi}_A$ begins to roll with a time-dependent frequency $\omega^2_\pm$ in the range of $0 < k < a|\phi_A|f$, one of the helicity would be negative leading to an gauge amplification from tachyonic fields where extra energy has been carried away by GWs. The time-dependent frequency $\omega^2_\pm$ is given from [2],

$$\omega^2_\pm(k,\tau) = \begin{cases} \omega^2_+(k,\tau), & k^2 + \frac{a}{f}\phi' \\ \omega^2_-(k,\tau), & (-)k^2 + \frac{a}{f}\phi' \end{cases} \begin{cases} non-tachyonic \\ tachyonic \end{cases} \tag{13}$$

V. Gravitational wave spectrum –
Stated in [44], the homogeneous wave equation of the gravity can be sated with reference to the wave vector $\vec{k}$ given as,

$$-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2[(h_{\alpha\beta})(\mathbf{x})] = -\frac{16\pi G}{c^4}T_{\alpha\beta}$$

$$[(h_{\alpha\beta})(\mathbf{x})] = a_{\alpha\beta}\, e^{i[\vec{k}\cdot\vec{x}-\omega t]} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{i[\vec{k}\cdot\vec{x}-\omega t]} \tag{14}$$

Since the gauge fields are the source of the SGWB which can be given by the metric perturbation $h_{\alpha\beta}$, the defined metric in the FLRW-flat universe could be given by,

$$ds^2 = -dt^2 + a^2(\delta_{\alpha\beta} + h_{\alpha\beta})dx^\alpha dx^\beta \tag{15}$$

Since gravity is highly dominated by dark photon, the quantity $\Psi_{\alpha\beta}$ as appeared in equation (9) can be modified to get $\widehat{\Psi}_{\alpha\beta}(\boldsymbol{k},\tau)$ is stated by the equation,

$$\widehat{\Psi}_{\alpha\beta}(\boldsymbol{k},\tau) = -\frac{\Lambda^{kl}_{\alpha\beta}}{a^2}\int \frac{d^3q}{(2\pi)^3}\left[\left(v'_\lambda(q,\tau)\varepsilon^\lambda_\alpha(\boldsymbol{q})\hat{a}_q\right) + \left(\lambda\, q v_\lambda(q,\tau)\varepsilon^\lambda_\alpha(\boldsymbol{q})\hat{a}_q\right)\right] \tag{16}$$

Where the 1st quantity in the third bracket denote the electric field, while the 2nd quantity denote the magnetic field, the solution of $h_{\alpha\beta}$ in terms of the reduced Plank's mass $\widetilde{P_m}$ and the modified dark photon operator $\widehat{\Psi}_{\alpha\beta}(\boldsymbol{k},\tau)$ is given by [2],

$$\hat{h}_{\alpha\beta}(k,\tau) = \frac{2}{a\widetilde{P_m}^2}\int_{\tau_{osc}}^{\tau} d\tau' a(\tau')\sin\frac{k(\tau-\tau')}{k}(k,\tau,\tau')\,\widehat{\Psi}_{\alpha\beta}(\boldsymbol{k},\tau') \tag{17}$$

The peak amplitude of the GWs is estimated at [1],

$$\frac{k^2}{a^2_{em}}h_{\alpha\beta}(t_{em}) \sim \frac{\rho_{src}(t_{em})}{\widetilde{P_m}^2} \sim \left(\frac{mf\emptyset}{\widetilde{P_m}}\right)^2\left(\frac{a_{osc}}{a_{em}}\right)^3 \tag{18}$$

Where the emission scale is a_{em} , source field energy density is ρ_{src} which is comparable to the axionic homogeneous mode temperature t_{em} as a function of the perturbed gravity metric $h_{\alpha\beta}$.

As pointed out in [1, 2, 3, 44], the SGWB can be best described as a superposition of incoherent sources. To parameterize the background strength vs. frequency wave f_{grav} by the energy density taken per unit logarithm of the present day, critical stress-energy density, given by, $\rho_{crit} = 3H_0^2c^2/8\pi G$ with the value of the Hubble constant H_0 as $70.5\ km/sec/Mpc$ [45], the resultant action can be given by,

$$\Omega_{GW}(f_{grav}) = \frac{1}{\rho_{crit}}\frac{d\rho_{GW}}{d\,\mathrm{In}(f_{grav})} \tag{19}$$

With the spectral density given by [46],

$$S_{GW} = \frac{3H_0^2}{10\pi^2}f^{-3}\Omega(f) \tag{20}$$

It is to be noted as in [44], that because of the cosmic microwave background radiation (CMBR), the GWs would be highly redshifted due to the expansion of the universe and to a greater degree, would be matter decoupled from earlier times. A more convenient form can be written as in [47], with the value of $h_{100} = H_0/\left(100\frac{km}{sec}/Mpc\right)$ as,

$$h(f) \equiv \sqrt{[S_{GW}(f_{grav})]} = (5.6\ \times 10^{-22})h_{100}\sqrt{\Omega(f)}\left(\frac{100\text{ Hz}}{f}\right)^{3/2}\text{Hz}^{-\frac{1}{2}} \tag{21}$$

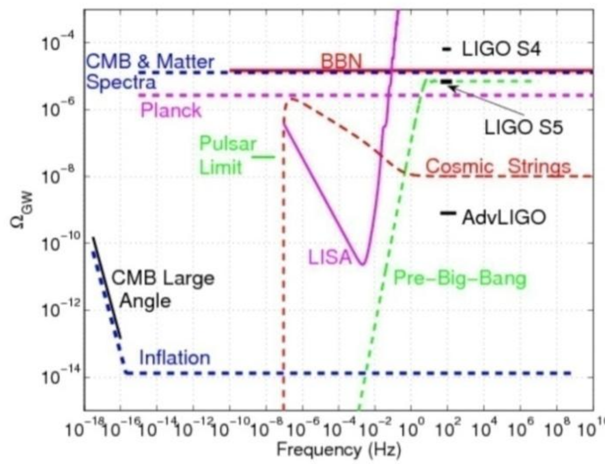

Figure 1: Different models and measurement of the Stochastic gravitational wave (SGW) are given [44, 48] which shows the results of S4 and S5 searches of LIGO have a frequency band around 100 Hz. The dashed curves are denoted by the matter spectrum over the integral of $\Omega_{GW}(f)$ over the frequency bands. WMAP CMBR and LISA detector are also shown for SGW. The SGW from pulsar is also shown corresponding to the fluctuations arrives at milliseconds intervals at around 10^{-8} Hz. CMBR as seen in large wavelengths (small frequencies) due to the possible redshifted photons are shown here. For e.g, the axionic/ALP gravity wave production at the early epoch of the inflammatory universe, cosmic strings etcetera.

From [1, 49], The GW frequency related to the axion mass at present day is given by,

$$v = \frac{k}{2\pi a_0} \sim 0.1\text{nHz}\frac{g^{1/4}_{*osc}}{g^{1/3}_{*Sosc}}\frac{k}{a_{osc}m}\left(\frac{\text{m}}{10^{-14}\text{eV}}\right)^{1/2} \tag{22}$$

With a_0 is the present scale parameter, $g^{1/3}_{*Sosc}$ is the effective relative density of the entropy perturbations at the onset of the oscillation. For $f\sim 10^{16}\ GeV$ the axion having mass $\sim 10^{-14}\ eV$ provides the DM relic in nHz range with $\Omega_{GW}(h)^2 \sim 10^{-15}$ orders of magnitude.

VI. Detecting gravitational waves –
As SGW arises from a superposition of incoherent sources, thereby it is difficult to detect such waves. The initial factor to determine a clear spectra of GWs is to separate it from the detectable noises and with a single detector, it is challenging to probe the astrophysical strain noise and eliminate it from the source waves. Nonetheless, with a good sensitivity and cross-correlations among different detectors, the incoherent integration of the correlated waves for a long period of time, an established technique can be made to search for SGWs [44]. For a successful detection, as mentioned in [44, 47], the scopes includes,

- Stationary over the measurement time T.
- Gaussian.
- Detectors with an uncorrelation among them.
- Uncorrelated with the SGW signals.
- Much greater in frequency above the SGW background.

The satisfying equation for time T, expectation value μ_y of Y with a coefficient relation to equation (20) given as [44],

$$\mu_y \equiv \langle Y\rangle = \frac{T}{2}\int_{-\infty}^{+\infty} df\ \gamma(|f|)S_{GW}(|f|)\tilde{Q}(f) \tag{22}$$

With $\tilde{Q}(f)$ being the Fourier transform of $Q(t)$ and $\gamma(|f|)$ [real] known as overlap reduction function [50] that characterizes, the sensitivity reduction from the isotropic stochastic background over the large separation and relative orientation of detectors, the separation time delay for a clear-cut waves.

The noise strain power of the GWs spectrum as given in [13],

$$\Omega_{\text{noise}} = \frac{2\pi^2 f^3 S_{noise}}{(3H_0^2)^2} \tag{23}$$

Where the quantity S_{noise} consists of the intrinsic noise spectrum and other astrophysical confusion noise, given as [13, 51],

$$S_{noise}(f) = S_{ins}(f) + S_{gen}(f) \tag{24}$$

Which varies over the nature of different detectors, the signal to noise ratio Θ is given by [13, 46],

$$\Theta^2 = N\, t_{obs} \int_{f_{min}}^{f_{max}} df \left[\frac{\Omega_{signal}(f)^2}{\Omega_{noise}(f)^2}\right]^2 \tag{25}$$

Where t_{obs} is the observing time of the experiment, $N = 1,2$ that performs the auto-correlation to measure the SGW background with f_{max} and f_{min} is the minimum and maximum possible frequency accessible to the detector with different space based and ground based interferometers like LISA [52], DECIGO [53, 54], BBO [55, 56, 57], Einstein Telescope (ET) [58, 59], Cosmic Explorer (CE), Hanford and Livingston [60].

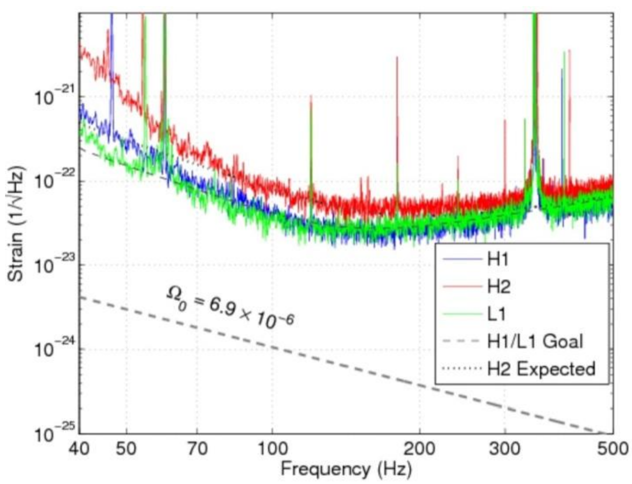

Figure 2: Of the best isotropic stochastic sensitivity, the limits on Ω_0 have been produced on successive runs of LIGO, with the time-run exactly frequency based sensitivity as 6.5 * 10^{-5} in the S4 data [61] to 6.9 * 10^{-6} in the S5 data [48] in the figure (from [44]) showing the superimposed scale of the S4 and S5 data along with the observable background dependent expectation ranges derived from the measurements with the strain amplitude spectral noise (relative to the S5 runs) having the upper limit ($\sim 100 \times lower$) placed on a SGWB through the cross-correlating functions.

As mentioned in [44], if the GWs detectors are co-located and co-aligned (for e.g., H1 and H2), then $\gamma(f) = +1$ in equation (22) for all frequencies, but for separated and co-aligned this norms to the limit $\gamma(f) \to +1$ and $f \to 0$ where the variance in the detectors pair sensitive to SGWB could be determined by,

$$\sigma_Y^2 \equiv <(Y - <Y>)^2> \approx \frac{T}{4}\int_{-\infty}^{+\infty} df\, P_1(|f|)\left(K\left(\frac{\gamma(|f|)S_{GW}(|f|)}{P_1(|f|)P_2(|f|)}\right)^2\right)P_2(|f|) \tag{26}$$

Where, K is an affine valued proportionality constant that arises because of the Fourier mode proportionality term $\tilde{Q}(f) \propto \frac{\gamma(|f|)S_{GW}(|f|)}{P_1(|f|)P_2(|f|)}$ with the $P_1(|f|)$ and $P_2(|f|)$ Is the strain noise frequency of the two detectors H1 and H2 respectively.

VII. Discussions –

We have shown implicitly that from the PQ phase transition, and CP symmetry breaking the axions (or ALP) originates and they goes through a hectic phase of kinetic mixing and the resultant couple of dark photons with the GWs and also the production of the GWs from the large scale catastrophic tachyonic instabilities that prevails in the early inflammatory stage of the universe. The axion potential and its gauge field connection in the form of various hidden sectors are stated which bifurcates into various SGWB. Analysis have been done on the spectrum of the GWs over the Hubble parameter and the cosmological redfshift due to the continuous expansion of the universe. This clearly states the possibility of the GWs productions on Nano-Hz range from axion/ALP through the circular polarizations of the tachyonic modes. Finally, the detection and analyze(ing) of the GWs with particular reference to SGWBs have been done and the resulting correlation function, to decrease the spectral noise from the detected GWs as an incoherent source properly visualized with the strain data of the isotropic stochastic sensitivity of the S4 and S5 run-datas.

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