Output Only Modal Analysis Using SVD

Pankaj Kumar Saini and Sharad Goyal

October 26, 2020

OUTPUT ONLY MODAL ANALYSIS USING SVD

Pankaj Kumar Saini and Sharad Goyal

Mit2020117@iiita.ac.inMit2020119@iiita.ac.in

Indian Institute of Information Technology, Allahabad

Abstract

Output only modal analysis is a technique where we do not need to measure the input excitations and just responses are measured. We only assume input excitation to be white noise. It is known as operational modal analysis because it deals with testing of structure under normal operations. This analysis is used in various types of engineering branches. It is the study of the dynamic properties of systems in the frequency domain. In this paper we define Singular value decomposition and its need in output only modal analysis. After we describe the process of analysis using singular value decomposition. We discuss its real-world applications and its future scope.

Keyword: **-** singular value decomposition, experimental transfer function, transfer function, residual effects.

1. Introduction

When the modal density is high, better results can be obtained by using the singular value decomposition to help separate the modes before the modal identification process begins. In a typical calculation, the matrix is formed of the transfer function data for a single frequency with each column representing a different drive point. The input is given to the singular value decomposition algorithm and left and right singular vectors and a diagonal singular value matrix are computed. The calculation again repeated at each analysis frequency and the resulting data is used to identify the modal parameters. In the optimal situation, the singular value decomposition will completely separate the modes from each other, so that a single transfer function is produces for each mode with no residual effects.

In practice, the modal transfer functions are never completely free from residual effects of nearby modes, but the resonance frequencies and damping loss factors can be accurately identified using simple one-degree-of-freedom models nonetheless. As an example, figure 1 shows a plot of the singular values as a function of frequency for a typical case, because the singular values are computed and output in order of descending magnitude, a single curve on the plot does not track a single mode. For example, just below 55Hz, the top two curves switch the modes that they’re tracking. However, by using the singular value decomposition at one frequency to decompose the coefficient matrix at nearby frequencies, it is possible to force the singular values to track only a single mode.

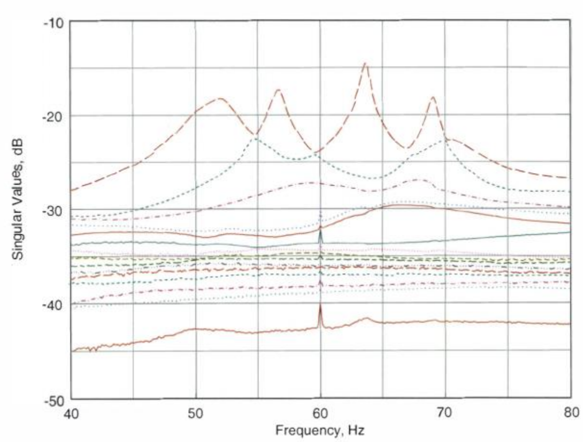


Figure 1. Singular values as a function of frequency for a series of closely-spaced modes

Modal analysis:

Modal analysis is a process of extracting modal parameters (natural frequencies, damping loss factors and modal constants) from measured vibration data. Since the measured data can be in the form of either frequency response functions or of impulse responses, there are frequency domain modal analysis and time domain modal analysis.

The fundamental of modal analysis using measured frequency response function data is about curving fitting the data using a predefined mathematical model of the measured structure.

Modern day experimental modal analysis systems are composed of

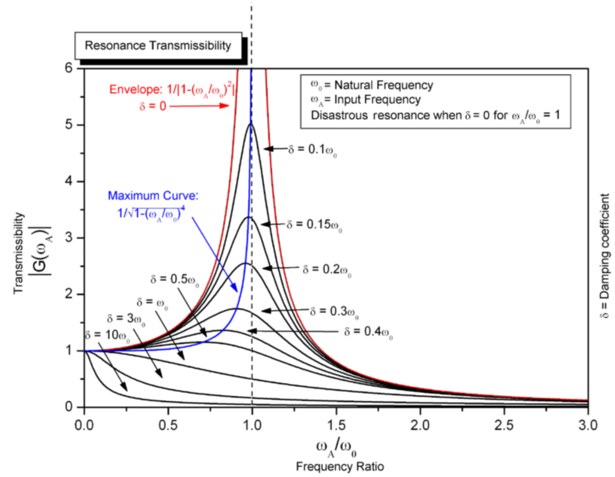
1) sensors such as transducers (typically accelerometers, load cells), or non-contact via a Laser vibrometer, or stereophotogrammateric cameras.

2) data acquisition system and an analog-to-digital converter front end (to digitize analog instrumentation signals).

3) host PC (personal computer) to view the data and analyze it.

Typical excitation signals can be classed as impulse, broadband, swept sine, chirp, and possibly others. Each has its own advantages and disadvantages.

The analysis of the signals typically relies on Fourier analysis. The resulting transfer function will show one or more resonances, whose characteristic mass, frequency and damping can be estimated from the measurements.



Need of output only Modal analysis:

For large engineering structures like bridges, dams and high-rise buildings, it is difficult to excite them artificially and to measure the excitations. Random forces act them such as on bridge different forces such as vehicle trafficking, wind excitation, ocean waves etc. excite the structure together so it's almost impossible to measure all these forces simultaneously. If the forces are not measured correctly, then modal analysis can't give accurate estimates of the modal parameters. On the other hand, when we excite such structure, large amplitude force is required to vibrate the structure at all points under observation. Such force can cause local damages to the structure. Natural conditions under which structure operates are very difficult to be produced in the laboratory. So, output only modal analysis is the best option available in these conditions as it depends on the responses only and the responses could be measured with high accuracy.

There are several examples where a prior accurate modal analysis could have prevented loss of lives and property. Some famous ones include:

Tacoma Narrows Bridge Disaster of 1940

The Tacoma Narrows Bridge was built in the state of Washington (USA). On November 7, 1940, at around 11 a.m., the bridge came down instantaneously. A later investigation revealed that the cause of the collapse was aeroelastic flutter.

Mexico City Earthquake of 1985

Another real-life example was the 1985 earthquake in Mexico City. The energy released during this earthquake was equivalent to 1114 nuclear detonations, and the earthquake was felt as far as Los Angeles, which is over 800,000 km away. Up to the 1950s, no earthquake codes existed.

Taipei 101 and Burj Khalifa

A real-life example can be seen in today's skyscrapers like Taipei 101 in Tokyo (Japan) or Burj Khalifa in Dubai (UAE). These megastructures use tuned mass dampers to absorb the energy and dampen the oscillations of the structures.

Singular value decomposition:

Singular value decomposition takes a rectangular matrix of gene expression data (defined as A, where A is a *n* x *p* matrix) in which the *n* rows represents the genes, and the *p* columns represents the experimental conditions. The SVD theorem states:

$$\mathbf{A}_{nxp} = \mathbf{U}_{nxn}\,\mathbf{S}_{nxp}\,\mathbf{V}^{\mathbf{T}}_{pxp}$$

Where

$\mathbf{U}^T\mathbf{U} = \mathbf{I}_{nxn}$

$\mathbf{V}^T\mathbf{V} = \mathbf{I}_{pxp}$ (i.e. U and V are orthogonal)

Where the columns of U are the left singular vectors (*gene coefficient vectors*); S (the same dimensions as *A*) has singular values and is diagonal (*mode amplitudes*); and V^T has rows that are the right singular vectors (*expression level vectors*). The SVD represents an expansion of the original data in a coordinate system where the covariance matrix is diagonal.

Calculating the SVD consists of finding the eigenvalues and eigenvectors of AA^T and A^TA. The eigenvectors of A^TA make up the columns of V , the eigenvectors of AA^T make up the columns of U. Also, the singular values in **S** are square roots of eigenvalues from AA^T or A^TA. The singular values are the diagonal entries of the S matrix and are arranged in descending order. The singular values are always real numbers. If the matrix A is a real matrix, then U and V are also real.

A. Finding the Resonances

Because it is easy to search a string of numbers for a peak, it may not seem like the process of identifying modal peaks should be especially difficult. However, it is undoubtedly the most difficult part of the modal identification process because the noise is always present in the transfer function measurements, even for carefully controlled systems. This is primarily a consequence of the uniform frequency-spacing required by the FFT algorithms. One way to avoid this difficulty is to require the user identify the locations of the modes by hand, usually with some sort of graphical interface, which is both tedious and time consuming.

In the mode finding algorithm, a number of enhancements have been implemented to make the process somewhat immune to noise. The first important way to reduce noise levels is to pass the data through the SVD algorithm. The output from the singular value decomposition consists of three matrices U, V and S. the U and V matrices are unitary (i.e. U U^H = 1, where the superscript H indicates a Hermitian transpose), and the S matrix contains the singular values on its diagonal and is real-valued. The three matrices from a decomposition of the original matrix as

$$\mathbf{H}(\omega_0) = \mathbf{U}_0\,\mathbf{S}_0\,\mathbf{V}_0^{\mathrm{H}}.$$

A plot of the singular values verses frequency for a typical example was in figure 1. We note that the topmost curve has the lowest noise levels and the bottommost curve has the largest, providing some confirmation that the singular value decomposition helps to reduce noise levels (assuming, of course, that we are primarily interested in the top few curves). As discussed in the previous section, the singular values are output in order of decreasing magnitude, so that they switch the modes they're tracking whenever two singular values cross. This problem is avoided by computing modal transfer functions, which force the singular values to track a single mode. The model transfer functions are computed by

$$\overline{\mathbf{S}}(\omega) = \mathbf{U}_0^H \, \mathbf{H}(\omega) \, \mathbf{V}_0 \ .$$

using the singular value decomposition at an initial frequency to decompose the transfer function matrix at nearby frequency as

The overbar on the matrix S indicates that it is no longer real-values or diagonal. At the initial frequency, the modal transfer function yields S0

$$\mathbf{U}_0^H \, \mathbf{H}(\omega_0) \, \mathbf{V}_0 = \mathbf{U}_0^H \, \mathbf{U}_0 \mathbf{S}_0 \mathbf{V}_0^H \, \mathbf{V}_0 = \mathbf{I} \, \mathbf{S}_0 \mathbf{I} = \mathbf{S}_0 \,.$$

because pre-multiplying by UH0 and post-multiplying by V0 yields.

A plot of the modal transfer functions verses frequency was given in figure 2. Even after passing the data through the SVD algorithms, it is not uncommon for the noise levels in the modal

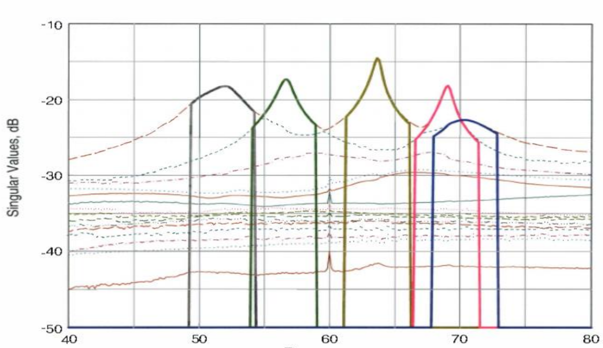


Figure 2. Modal transfer functions overlaid on the singular values as a function of frequency

transfer function data to be too high for a simple peak finding algorithm to work reliably.

To further reduce the noise levels in the modal transfer function data, it is passed through a smoothing filter before searching for peaks in the response. The parameter for the smoothing filter were chosen such that the peaks in the response do not shift significantly during the smoothing

$$\mathrm{H}(\omega) = \frac{\mathrm{A}}{\omega_\mu^2 - \omega^2 - i\,\eta_\mu\,\omega_\mu^2} \ .$$

process. this requires the filter to preserve higher order moments, yielding results with minimal changes in the height and width of the peaks. A more thorough discussion is given in the reference. the peaks in the response still can shift frequencies and may even shift as the input data to the smoothing filter changes. This means that the same resonance may possibly be identified several times, thus requiring a method for finding and eliminating duplicate modes.

Fortunately, it is relatively easy to detect duplicate modes using the modal assurance criteria (MAC). First, we must rotate the modal transfer functions so that the resonance frequency occurs at a consistent phase angle. The different phase rotations occur because the singular value decomposition yields real singular values, such that zero phase is always referenced to the frequency used to generate the modal transfer functions. We sill assume a single degree-of-freedom representation of the mode such that the modal transfer function can be represented as

As we noted previously, the SVD algorithm reduces noise in the largest singular values at the expense of the lowest singular values, which is desirable because most of the relevant modal peaks occur in the few largest singular values. Thus, the lowest singular values have relatively high noise levels and do not contain relevant information. To avoid identifying numerous extraneous peaks in the lowest singular values, the user can choose to apply the peak finding algorithm to only a few of the largest singular values.

B. Modal Parameter Identification

The last step in the process is to actually determine the resonance frequencies and loss factors from the modal data. There are many methods for computing the modal parameters once the data for a single mode has been isolated. As for the circle fit algorithm, a weighting function is applied before solving the equation system to that the data points near the resonance peak are more heavily weighted. Once the resonance frequency and damping loss factor have been determined, the modal transfer function can be synthesized and compared to the input data to assess the accuracy of the fit. To try to make the predictions more reliable and immune to noise, these calculations are performed for different average error is used for the predictions. If the modal transfer functions are relatively free of noise, the fit is typically better using only a few points near the peak, otherwise, better results are obtained using more data.

C. Testing the Algorithm

To test the curve-fitting algorithm, a finite element model was used to generate simulated experimental data with known resonance frequencies and damping loss factors. Along with our algorithm, several methods for computing damping in X-Modal were also tested including rational fraction polynomials and the complex mode indicator function (CMIF).

D. Step-By-Step Analysis Procedure

The following steps will yield predictions for the resonance frequencies and damping loss factors from a set of transfer function data. The optimal steps should be performed if mode shapes are to be computed as well.

1. Convert the experimental transfer function data to uff(universal file format) for dataset. If the data is in vna file format, the MATLAB program UFF can be used to perform the conversion.
2. (optional) generate a geometry file representing the surface locations where the transfer function data was taken. It should be in the standard format for input files to the boundary element program POWER and should be named.
3. (optional) generate an in.txt file, which contains input values for the parameters.
4. Run the program CONV_EXPERIMENTAL to automatically identify the resonance frequencies, damping loss factors, and (possibly) mode shapes. As mentioned in the text, a number of extraneous modes are inevitably identified.

   NOTE: if program crushes because it finds too many modes, the parameter NP file should be reduced such that fewer singular values are searched for peaks during the automatic identification process.

7. Application in Real World

1. Structural Engineering:

   Output only modal analysis uses the overall mass and stiffness of a structure to find the various periods at which it will naturally resonate. These periods of vibration are very important to note in earthquake engineering, as it is imperative that a building's natural frequency does not match the frequency of expected earthquakes in the region in which the building is to be constructed. If a structure's natural frequency matches an earthquake's frequency, the structure may continue to resonate and experience structural damage.

Modal analysis is also important in structures such as bridges where the engineer should attempt to keep the natural frequencies away from the frequencies of people walking on the bridge. This may not be possible and for this reason when groups of people are to walk along a bridge, for example a group of soldiers, the recommendation that they break their step to avoid possibly significant excitation frequencies.

Electrodynamics:

Rotating machines play a vital role in modern economics. Most industrial processes where energy is processed are based on rotating machinery. Thus, it is increasingly important to maintain those machines in the good technical state. Main drivers for final users are:

– avoidance of catastrophically failures,

– decrease of maintenance costs,

– increase of availability.

This need, in turn, create strong demand for diagnostic techniques.

Future Scope:

1. Theoretical works are performed since decades, starting from simplified, linear rotor models. With advances in rotordynamics research new processes were identified and described.
2. The OMA results can be adopted for a wide range of significant applications. One of the most important applications of OMA is finite element (FE) model updating. The accuracy of FE model is based on the initial assumptions about material properties, boundary conditions and geometry of the structure. Choosing the accurate and real mentioned assumptions is a big challenge, particularly for the structures about which there isn't certain knowledge like historic buildings. As a consequent, the experimental results should be utilized to update, verify or optimize the initial FEM model which can present more reliable behaviour of the structure.

8. Conclusion

Rapid development and the increasing popularity of operational modal analysis (Output only modal analysis), justify the need for more comprehensive review studies. This paper widely reviewed theoretical and practical aspects involved in role of Singular value decomposition of OMA. In output modal analysis there can be many techniques to extract modal parameters but SVD reduces the redundant modes and help these techniques to work efficiently. SVD is a prior step to Principal Component analysis and can also be used to find pseudoinverses of the matrix.

9. References

[1] https://www.researchgate.net/publication/235028460_Modal_Analysis_Using_the_Singular_Value_Decomposition

[2] https://en.wikipedia.org/wiki/Modal_analysis

[3] https://www.simscale.com/blog/2016/12/what-is-modal-analysis/

[4] https://www.diva-portal.org/smash/get/diva2:829819/FULLTEXT01.pdf

[5] https://medium.com/@jonathan_hui/machine-learning-singular-value-decomposition-svd-principal-component-analysis-pca-1d45e885e491

[6] https://www.sciencedirect.com/science/article/pii/S1877705814030537