Suprathreshold Stochastic Resonance for Gamma Noise with Watermarking Application

Sumit Kumar, Nancy Chauhan and Rajib Kumar Jha

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Suprathreshold Stochastic Resonance Characterization for Gamma Noise with Watermarking Application

Sumit Kumar[1], *IEEE, Member*, Nancy Chauhan [2], Rajib Kumar Jha [1], *IEEE, Senior Member*
[1] Indian Institute of Technology Patna, [2] Indian Institute of Information Technology Una
Email:(sumitphd13,nancychn1,jharajib)@gmail.com

***Abstract*—Noise plays a constructive role in a lot of non-linear applications. Many non-linear systems perform better when some calculated external noise is added. This phenomenon is called stochastic resonance (SR). When a parallel array of SR is used, it is termed as Suprathreshold Stochastic Resonance (SSR). Many of the systems and models where SR is effectively observed are non-linear systems with a single threshold value.**

In this paper, the effect of SSR using Gamma noise has been reported, and the expression for cross-correlation has been derived. Furthermore, the same concept has been used in a watermarking application, where Gamma noise is added as the signature in the cover image. Gamma noise is later added externally to the watermarked image which maximizes the cross-correlation at a particular value of the variance of the SR noise. The obtained results suggest that the maximum value of cross-correlation is attained for a certain amount of added noise.

***Index Terms*—SSR, Watermarking, Gamma noise.**

I. Introduction

Stochastic resonance (SR) is a phenomenon that is said to occur when the presence of noise benefits the optimal output of some non-linear systems. The concept of SR was proposed by Benzi *et al.* [1], where SR was loosely discussed as an event in which the presence of an optimal amount of noise leads to an increment in signal-to-noise ratio.

Collins et al. [2] discuss the FitzHugh-Nagumo neuron model characterized by aperiodic stochastic resonance (ASR). They mention a measure based on the cross-correlation coefficient to characterize ASR. A lot of work including the study of leaky integrate-and-fire and Hodgkin-Huxley neuron models has been done using SR with a cross-correlation measure to find the optimal noise level for a considerable number of neurons [3]. SSR has also been demonstrated in integrate-and-fire neurons in the context of both input correlations and for other noise-based enhancement effects(cite Durrant et al. 2011) [4], along with saturation [5]. Furthermore, SSR concept has also been used for SR in visual perception enhancement, and the findings on using a particular noise have also been discussed [6].

Stocks et al. [7] show that SR can occur where Shannon's average mutual information measure between input and output of the array gets optimized under the condition that all thresholds are set to be the same value. Most importantly, SR is said to have happened regardless of whether the input signal is entirely subthreshold or not. This is the specialty of SSR which justifies the name "Supra-threshold". Similarly, more work has been shown in McDonnell et al. [8], Kumar et al. [9]–[11]. In this paper, the concept of SSR [12] has been discussed, and the expression for the cross-correlation coefficient for Gamma signal and Gamma noise distribution has been derived. Subsequently, the same concept is applied to watermark signal detection. [13]

The paper is organized as follows. In Section II, the expression for the cross-correlation coefficient for Gamma distributed signal and Gamma distributed noise has been derived. Section III deals with proposed method for watermark embedding and extraction. The results are discussed in Section IV. Finally, the conclusion of the paper is given in Section V.

II. Basic Mathematics

Here, the exact analytical expression of the cross-correlation coefficient for Gamma noise has been derived. Suprathreshold stochastic resonance (SSR) was described by Stocks et al. [14].

A. Mathematical Model for SSR and Single Threshold

In this method, the output depends on the threshold value *i.e.,* it becomes one if input signal X and added noise ζ crosses the threshold Δ, otherwise it becomes zero. Mathemtically, the thresholded signal Y_i is given as

$$Y_i = \begin{cases} 1 & \text{for} \quad X + \zeta_i > \Delta_i \\ -1 & \text{for} \quad X + \zeta_i < \Delta_i \end{cases} \tag{1}$$

Considering Y_{norm}:=$\frac{1}{N}\sum_{i=1}^{N} Y_i$. The normalized signal Y_{norm} becomes

$$Y_{norm} = \frac{1}{N}\sum_{i=1}^{N} sgn(X + \zeta_i), \tag{2}$$

where $sgn(.)$ is sign function.

B. Calculation of correlation

The probability that the output of any thresholded signal is 1 (*i.e.*, the sum of signal and noise is greater than Δ_i) is denoted by $P_{1|x}$ for a known signal, x. Here, it is assumed that the threshold value is considered as θ and the noise distribution is identical. So, $P_{1|x}$ is given as follows.

$$P_{1|x} = \begin{cases} \int_0^{\infty} f_\zeta(\omega) d(\omega), & \text{continuous } \zeta \\ \sum_{k=\theta-x}^{\infty} P(\zeta = k), & \text{discrete } \zeta. \end{cases} \tag{3}$$

where $f_\zeta(\zeta)$ is the pdf of the noise. Now, we want to calculate E[$Y_{norm}|X = x$]. When Y_{norm}:=$\frac{1}{N}\sum_{i=1}^{N} Y_i$ from Eq.2, the expected value of Y_{norm} given x is

$$E[Y_{norm}|X] = \frac{1}{n} E\left[\sum_{i=1}^{n} sgn(X + \zeta_i)\right] \tag{4}$$

Since all noises are independent and identically distributed, so

$$E[Y_{norm}|X] = E[sgn(X+\zeta_i)] \quad (5)$$

$$E[Y_{norm}|X] = (1P_{1|x} + (-1)(1-P_{1|x})) = (2P_{1|x}-1) \quad (6)$$

The cross-correlation (ρ) between X and Y_{norm} can be found by

$$\rho = \frac{E[XY_{norm}] - E[X]E[Y_{norm}]}{\sqrt{var[X]var[Y_{norm}]}} \quad (7)$$

$$E[xY_{norm}] = \int_{-\infty}^{\infty} xE[Y_{norm}|X]P(x)d(x). \quad (8)$$

$$E[Y_{norm}|X] = 2\int_{-\infty}^{\infty} xP(x)P_{1|x}dx - E[x] \quad (9)$$

For discrete x the integral in above expression will be replaced by summation over the entire domain of x.

C. *Correlation for Gamma Distributed Signal*

The cross-correlation is computed between Y_{norm} and x. The pdf of watermark signal taken as Gamma distributed noise is $P_x(.)$. The pdf of the random noise which is used in detection of embedded watermark signal is also Gamma distributed and is given by $P_\zeta(.)$. So,

$$f_x(x) = \begin{cases} \frac{1}{\Gamma(a)}e^{-x}x^{a-1} & , \quad 0 \le x \le \infty, a > 0 \\ 0 & , \quad elsewhere \end{cases} \quad (10)$$

$$f_\zeta(\zeta) = \begin{cases} \frac{1}{\Gamma(b)}e^{-\zeta}\zeta^{b-1} & , \quad 0 \le \zeta \le \infty, b > 0 \\ 0 & , \quad elsewhere \end{cases} \quad (11)$$

$$P_{1|x} = \int_{\theta-x}^{\infty} \frac{1}{\Gamma(b)}e^{-\zeta}\zeta^{b-1}d\zeta = \frac{1}{\Gamma(b)}\sum_{k=0}^{b-1} e^{-(\theta-k)}\frac{(\theta-x)^k}{k!} \quad (12)$$

The expected value of $P_{1|x}$ over the signal distribution can be given as below.

$$E[P_{1|x}] = \int_0^{\infty} P_{1|x}\frac{1}{\Gamma(a)}e^{-x}x^{a-1}dx = \frac{1}{\Gamma(a)}\frac{1}{\Gamma(b)}\sum_{k=0}^{b-1}\frac{e^{-\theta}}{k!}\int_0^{\theta}\left(1-\frac{x}{\theta}\right)^k\theta^k x^{a-1}dx, \quad (13)$$

where $\alpha = \int_0^\theta \left(1-\frac{x}{\theta}\right)^k \theta^k x^{a-1}dx$,

$$\alpha = \int_0^{\theta}\left(1-\frac{x}{\theta}\right)^k\left(\frac{x}{\theta}\right)^{a-1}dx. \quad (14)$$

Let $\frac{x}{\theta} = t$. Furthermore, it can be written as

$$\alpha = \theta^{k+a}\frac{\Gamma(k+1)\Gamma a}{\Gamma(k+a+1)} \quad (15)$$

Then, $E[P_{1|x}]$ can be written as follows.

$$E[P_{1|x}] = \frac{1}{\Gamma(b)}\sum_{k=0}^{b-1}\frac{e^{-\theta}\theta^{k+a}}{\Gamma(k+a+1)} \quad (16)$$

Now, substituting Eq. 16 in Eq. 18, we get

$$E[y|x] = E[\text{sgn}(x+\zeta_i)|x] = 2P_{1|x} - 1. \quad (17)$$

Similarly,

$$E[y] = E[E[y|x]] = 2E[P_{1|x} - 1] \quad (18)$$

Now we need to calculate $E[y^2|x]$.

$$E[y^2|x] = \frac{1}{N^2}\left[N + N(N-1)(2P_{1|x}-1)^2\right] \quad (19)$$

$$E[y^2] = \frac{1}{N} + \left(\frac{N-1}{N}\right)\left[4E\left[P_{1|x}^2\right] - 4E\left[P_{1|x}\right] + 1\right] \quad (20)$$

Similarly, we can calculate $E[xy]$ as below.

$$E[xy] = 2\int_0^{\infty}\left(xf_x(x)P_{1|x}\right)dx - E[x] \quad (21)$$

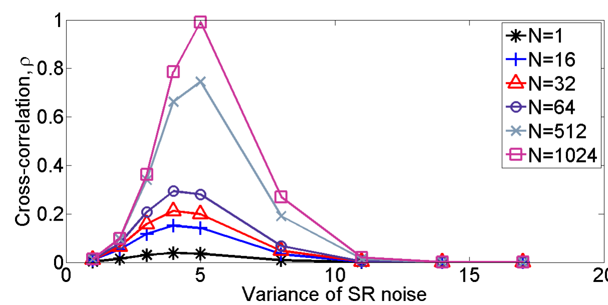


Fig. 1: Normalised correlation value for SR noise at different N.

The results clearly show that for increasing number of parallel arrays, the cross-correlation value ρ is increasing.

III. Proposed Method for Watermark Detection

The embedding and extraction steps are shown in Fig. 2 and Fig. 3 respectively.

A. *Watermark embedding*

Here, the watermark embedding technique has been discussed in detail. Initially, the original image is taken of size 256×256 as shown in Fig. 4. The size of the watermark signal is also taken to be 256×256. The algorithm used for watermark embedding is given below.

1) Three level DWT coefficient of LL bands are evaluated.
2) A watermark signal that is Gamma distributed is generated with a certain *seed* value.
3) $T1$ is initialized, and the watermark is embedded in those pixels whose intensity is higher than $T1$.
4) Finally, the inverse wavelet transform is taken to get the watermarked image. The size of the watermarked image is same as that of the original image.

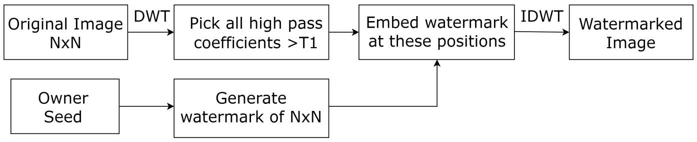


Fig. 2: Proposed method for watermark embedding

B. *The Proposed algorithm using SSR and single threshold*

For watermark detection, a 3-level DWT transform is applied to image. Significant coefficients are selected and correlated with the original watermark X. For the selection of DWT coefficients, a threshold $T_2 > T_1$ is chosen. This is done because correlation is calculated only for all those coefficients where watermarks were added at the time of embedding. Besides, external noise will have increased the values of these pixels and so, a threshold higher than T_1 is needed.

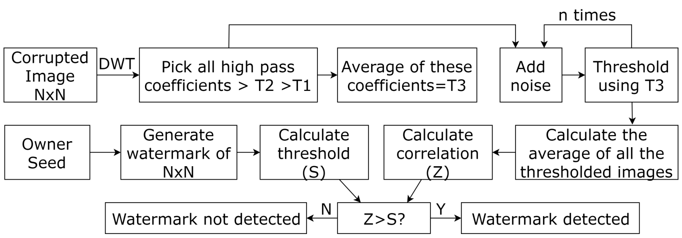


Fig. 3: Proposed method for watermark extraction

The proposed algorithm is discussed below.

1) A 3-levels DWT transform is done on the watermarked image to get V_d.
2) Gamma distributed noise is added to those DWT coefficients which are higher than the second threshold value T_2.
3) The third threshold value (say M) is taken as the mean of those DWT coefficients of the watermarked image which are higher than the second threshold value T_2. The noise added DWT coefficients(say U) is then compared with the third threshold value (M). If U is less than M, then the corresponding coefficient is replaced by zero; otherwise, it is replaced by 255. So, a thresholded image V_d' is obtained.
4) Similar to Step-1 and Step-2, Gamma distributed noises are added 30 times to the 3-level DWT coefficients of the distorted watermarked image. This is done because the addition of noise 30 times to the 3-level DWT coefficients improves the detector response. Increasing the number of times that noise is added further gives no improvement in detection and the detector response saturates. However, if noise is added less than 30 times to the 3-level DWT coefficients, the detector response is found to be deteriorating.
5) The 30 threshold images, $V_d'i$, are summed and averaged to get a single frame V_d''. The mathematical equation for this is given in Eq. 2, where N is the number of times the noise is added to the DWT coefficients. These are the resultant DWT coefficients of the SSR based signal threshold method.

$$V_d'' = \left[\frac{1}{30}\sum_{i=1}^{30} V_{di}'\right], \tag{22}$$

where $V_{di}' = Y_i$ and $V_d'' = Y$.

6) Correlation Z is calculated between V_d'' coefficients and original watermark X using Eq. 23, given below. x_i are the individual pixel values in the watermark X. The size of the V_d'' and X are the same as the original image size.

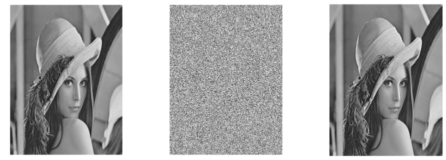
$$Z = \frac{1}{A}\sum_{i=1}^{M} V_{di}'' x_i, \tag{23}$$

where V_{di}'' is the suprathreshold stochastic resonance based image coefficients of the distorted image, x_i are the coefficients of the original watermark, i runs overall coefficients whose values are $> T_2 > T_1$, A is the number of such coefficients, M is the difference of size of the image to the size of LL band of the image, and Z is the correlation value.

7) A fourth threshold S is defined using Eq. 24 given below, where i runs over all coefficients with values $> T_2 > T_1$, A is the number of such coefficients, and S is the threshold value. The value of α has been taken as 0.2.

$$S = \frac{\alpha}{2A}\sum_{i=1}^{M} |V_{di}''| \tag{24}$$

8) Compare correlation value Z with the fourth threshold of S. Watermark is said to be detected in the image if $Z > S$. A higher value of the ratio of correlation to the threshold($\frac{Z}{S}$) provides more authenticity to the detection of watermark. It has been shown in Fig. 5.



(a) (b) (c)

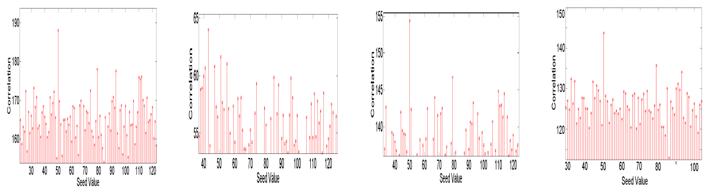
Fig. 4: (a) Original image, (b) Watermark signal, (c) Watermarked image

IV. Result & Discussion

In this section, the results of watermark detection application with the insertion of external noise are discussed. The software platform used was MATLAB 2014b with 8GB RAM and Intel(R) Core(TM)i3-4130 CPU @ 3.40 GHz. The watermark signal was embedded into the wavelet coefficients whose value were greater than $T1$. Since the value of $T1$ was taken such that $T1 > 0$, it may be said that it loosely followed a shifted version of Gamma distribution. Assuming that the signal is Gamma distributed, Gamma noise was selected as

TABLE I: Comparative correlations of proposed method with others for Lena image.

	Dugad et al. [15]	Jha et al. [16]	Barni et al. [17]	Hien et al. [18]	Hien et al. [19]	**Proposed method**
Correlation (Z)	32.88	84.1	0.816	3.5	1.62	**166.0408**
Threshold (S)	9.76	14.0	0.41	1.73	0.45	**16.4476**
Ratio (Z/S)	2.81	6.07	1.99	2.02	3.6	**10.0952**



(a) (b) (c) (d)

Fig. 5: Cross-correlation results in the presence of different attacks (a) Salt & pepper attack (b) Crop attack (c) Histogram equalization attack and (d) Gaussian noise(0.05)

SSR noise. In the Table I, the correlation (Z), threshold (S) and ratio ($\frac{Z}{S}$) for some of the other known techniques has been compared with the results for the proposed method. The proposed method gives us a correlation value (Z) that is significantly higher when compared with Dugad et al. [15] and Jha et al. [16]. It is also seen that the ratio between correlation coefficient and threshold ($\frac{Z}{S}$) is higher for the proposed method. So, it can be said that the proposed method works suitably well when compared to others.

V. Conclusion

A new method of using SSR with Gamma noise is developed, and the expression for cross-correlation has been derived. For different values of noise variance (*i.e.,* ζ), cross-correlation coefficient values have been plotted. The plot shows a maximum of cross-correlation coefficient at a particular value of σ of stochastic resonance noise. Here, b is the variance of the externally added Gamma noise. Furthermore, this concept has been applied to enhance watermark detection. The authenticity of watermark detection improves in the presence of SSR noise which confirms the benefits of SSR. SSR is tested on different attacked watermarked images, and the correlation is found to be 4-5 times higher than the threshold. This demonstrates that the SSR method which is noise based improves the correlation in watermark detection. The detector response proves the robustness of SSR based watermark detection technique.

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